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MODELS AND METHODOLOGY FOR LIFE CYCLE COST AND TEST AND EVALUATION ANALYSIS

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Office of the Assistant for Study Support

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This report documents various models and methodology which were developed during the course of some analytical studies on life cycle cost and test and evaluation. These studies were conducted by the Office of the Assistant for Study Support (OAS) at the request of DCS/Development Plans, Headquarters AFSC.

The objectives of the study were to: investigate the present methods of subsystem reliability specification and identify limitations associated with these methods; investigate new and innovative techniques for subsystem reliability

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management and identify benefits to be derived in terms of higher performance/lower costs; and, develop models and methodology applicable to life cycle cost and test and evaluation analyses.

A methodology was developed which relates system performance to the important parameters of life cycle costs such as subsystem reliability levels, cost of subsystem reliability improvement, and logistic support costs. A mathematical model was developed in which an optimization procedure is utilized to determine which subsystem reliabilities should be improved in order to obtain the required level of performance for the least cost. The model is quite general and can be applied to any system for which a mission profile can be defined and mission critical subsystems can be identified.

The methodology can be used early in the development program to negotiate initial requirements and evaluate competing subsystems. In addition, the methodology provides a very valuable management tool during system test and evaluation for continuous assessment of test results, and for identification of those subsystems requiring modification early in the program.

Finally, an appropriate measure of system effectiveness must be established and the relationship between the subsystems and system effectiveness must be determined. Then the model will identify those subsystem options which yield maximum system effectiveness for any level of total system cost.

Block 19:
Life Time Targets Hit
Probability of First Shot

FOREWORD

This technical report presents the detailed methodology of various modeling techniques developed by OAS during the course of analytical studies on life cycle costs and test and evaluation. These studies were initiated at the request of DCS/Development Plans, Headquarters AFSC and were designed to investigate new and innovative methods of reliability management and to develop models and methodology applicable to life cycle cost and test and evaluation analyses.

The authors wish to acknowledge the assistance of Mr. Donald C. Fronterhouse of AFWL/ADS in developing the computer programs for the models and to Capt Kenneth E. Hinkle of OAS for assistance in preparing the final report.

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This report has been reviewed and is approved for publication.

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EXECUTIVE SUMMARY

This technical report presents the detailed methodology of various modeling techniques developed by the Office of the Assistant for Study Support (OAS) during the course of analytical studies on life cycle costs and test and evaluation. These studies were initiated at the request of DCS/Development Plans, Headquarters AFSC and were designed to investigate new and innovative methods of reliability management and to develop models and methodology applicable to life cycle cost and test and evaluation analyses. The principal impetus in developing the models came from one of the major findings in the initial study performed by OAS on test and evaluation analysis. This finding concerned the generally poor field reliability experienced by the A-7D avionics systems. Further investigations showed that Air Force avionics systems generally experience poor reliability in the operational environment. There are a multitude of reasons for this; one reason being that current avionics systems are extremely complex and low reliability is therefore to be expected. This does not mean, however, that low reliability of avionics systems should be accepted as a way of life. In fact, every attempt should be made to achieve the highest reliability possible commensurate with the complexity of the system and cost constraints.

In probing deeper into reliability problems associated with Air Force avionics systems, one must examine the regulations, tests, and demonstrations that form the basis for Air Force acceptance and procurement of avionics systems. Air Force Regulation 80-5 states that "realistic and meaningful R&M characteristics and levels will be determined by cost effectiveness analysis, reflecting both system effectiveness and life cycle costs." AFR 80-5 states further that "the establishment of minimum acceptable R&M levels must be determined on the basis of realistic operational needs." Unfortunately, there has not been an effective technique for selecting levels of subsystem R&M which would optimize the total cost effectiveness of a system. Therefore, the desired system effectiveness is not being achieved, and life cycle costs are greatly exceeding desired levels.

A corollary problem has been the total lack of similarity between contractual R&M requirements and the values actually achieved in the field. Much of this problem is due to deficiencies in the test procedures used (normally from MIL-STD-781B for electronic equipment), but it is also affected by the lack of a rational approach to the establishment of requirements or for evaluating alternatives when testing indicates that the initial requirements are not being achieved. The result has been a spiral of higher and higher user requirements for which the developers have been expending more and more resources in a futile attempt to achieve. The models and methodology developed herein represent a first step in attempting to bridge this gap between initial requirements and achievable operational capabilities. In addition, the models and methodology can provide other information of interest to decision-makers concerned with either development, acquisition, testing, logistic support, or life cycle costs. The models are quite general and can be applied at various stages of system development. The models were developed in the context of a total system consisting of a number of subsystems. However, the models can be applied at the subsystem level by considering the total system to be the subsystem of interest consisting of its components.

The initial model is a Mission Completion Success Probability (MCSP) model. MCSP models are applied to show the dependence of mission success upon the aggregate of subsystems. MCSP models have not been used extensively in Air Force programs. In some instances where MCSP modeling techniques were employed, they involved complicated simulation methods. Generally, an MCSP model involving simulation does not readily lend itself to identification of critical subsystems, or to evaluation of critical subsystem improvement. The OAS MCSP model developed during this study is a generalized, probabilistic model. Using A-7D data, the utility of the model has been demonstrated by ranking subsystems according to abort causing failures and also in determining the MCSP enhancement due to improvements in individual subsystem reliability. The next step in developing the overall methodology is to consider reliability optimization, i.e., the tradeoff between levels of reliability and lifetime support cost to decrease system life cycle cost. MCSP models alone are inadequate for this task since they do not measure the impact of subsystem reliability levels on system life cycle cost. Combining reliability optimization with MCSP considerations leads to the development of the Designing to System Performance/Cost (DSPC) model.

The DSPC methodology represents a new and innovative approach to system acquisition, and preliminary results indicate that this technique will provide very valuable information to the decision-maker. This methodology systematically identifies those subsystem options which provide the highest system performance at any prescribed level of cost (either acquisition cost or acquisition plus logistic support cost). The DSPC model is compatible with designing to system cost, or performance, or both. Once total system reliability specifications are established, each individual subsystem has a corresponding installed reliability and cost goal which allows realistic and continuous evaluation and adjustment as the subsystem is developed to maturity.

Along with the DSPC methodology appropriate measures of effectiveness must be tailored to the particular mission of interest and related to system performance parameters. In this way the methodology can provide some of the many inputs the decision-maker requires. In this report two measures of effectiveness for fighter aircraft are presented. In the case of air-to-ground fighters, it is shown that an evaluation of the effectiveness must account for the interaction of availability, abort probability, kill potential, and survivability; and survivability is often the most dominant factor. For air-to-air fighters, the exchange ratio (Red Aircraft destroyed per Blue Aircraft destroyed) is an important measure of worth, and it can be expressed as a function of weapon effectiveness, maneuver capability, and first shot probability with first shot probability being the most important parameter.

As mentioned previously, the models and methodology can be applied at various stages of system development and were developed to augment established Air Force procedures. One of the more important applications of the models would be in providing information for the establishment of meaningful reliability requirements during the conceptual and validation phases. Another important application would be in employing the DSPC model during reliability validation tests. AFR 80-5 makes provisions for reliability evaluation tests, i.e., tests to determine reliability deficiencies rather than to demonstrate achievement of specified values. After identifying the reliability deficiencies in a given subsystem, there are various options available for taking corrective action such as redesign, use of higher quality components, redundancy, environmental protection, etc. Each of these options will have associated with it a certain reliability improvement along with the cost of achieving this improvement. The DPSC model applied to this subsystem would identify those corrective action options which would provide the highest performance at a prescribed cost.

In conclusion, it is useful to review briefly the stepwise procedures and inputs required for implementing the OAS analytical models. These procedures and inputs are as follows:

> Specify the mission profile by phases and the subsystem operating time during each phase.
>
> Identify the mission critical subsystems and specify their MTBFs.
>
> From failure modes effects analysis or other data determine the conditional probability of abort given failure.

With the above data, the mission completion success probability can be calculated and the subsystems ranked according to their probability of causing a mission abort. In addition, a sensitivity analysis can be performed to determine the increase in MCSP due to increasing the MTBF of any selected subsystem. Even without cost data the above information is useful for the planner early in the program in identifying the most troublesome subsystems and indicating those subsystems for which additional options are desired.

When options for the various subsystems are available and the acquisition cost of each subsystem option is estimated, the OAS model can optimize system performance over acquisition cost, i.e., for any level of system acquisition cost those options are identified which will yield maximum performance.

The next step is to obtain the average cost per repair for each subsystem option. Then the model can optimize over total system cost (acquisition plus logistic support cost).

Finally, an appropriate measure of system effectiveness must be established and the relationship between the subsystems and system effectiveness must be determined. Then the model will identify those subsystem options which yield maximum system effectiveness for any level of total system cost.

The models and methodology presented herein are just one approach to providing the decision-maker with important information. These models can be extended if more detailed analysis is required, and it is hoped that this methodology will provide some guidelines for other workers in developing and formulating models for their own particular applications.

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SECTION I
INTRODUCTION

1. GENERAL

During the past few years, the Office of the Assistant for Study Support (OAS) has been engaged in a variety of analyses concerning test and evaluation and life cycle costs (References 1 and 2), and in the course of these analyses, various mathematical models were developed. An overview of this approach to life cycle cost and test and evaluation analysis is presented in Reference 2.

For convenience of reference and in the belief that the models might be of use to other workers in the areas of test and evaluation and life cycle costs, the models are presented here along with detailed methodology and examples.

It should be noted that a complete life cycle cost model was not developed during the course of this study, but rather such things as subsystem reliability levels and logistic support cost and their impact on life cycle cost were analyzed. In addition, a generalized approach for relating system effectiveness to system life cycle cost is developed.

2. BACKGROUND

a. Operational Reliability Deficiencies. The principal impetus in developing the models came from one of the major findings in the initial study performed by OAS on test and evaluation analysis. This finding concerned the generally poor field reliability experienced by the A-7D avionics systems. Low operational reliability of sophisticated avionics equipment is not in itself an unexpected revelation, but the low reliabilities in conjunction with wide discrepancies between the established reliability requirements for the A-7D avionics systems and their respective operational reliability levels does appear to be significant. Further investigation showed that such discrepancies are not unique to the A-7D program but are also prevalent in other Air Force weapon systems.

b. Impact of Reliability Deficiencies on System Effectiveness and Life Cycle Costs. In probing deeper into reliability problems associated with Air Force avionics systems, one must examine the regulations, tests, and demonstrations that form the basis for Air Force acceptance and procurement of avionics systems. Air Force Regulation 80-5 states that "realistic and meaningful R&M characteristics and levels will be determined by cost effectiveness analysis, reflecting both system effectiveness and life cycle costs." AFR 80-5 states further that "the establishment of minimum acceptable R&M levels must be determined on the basis of realistic operational needs." Unfortunately, there has not been an effective technique for selecting levels of subsystem R&M which would optimize the total cost effectiveness of a system. Therefore, the desired system effectiveness is not being achieved (Figure 1), and life cycle costs (Figure 2) are greatly exceeding desired levels.

In Figure 1, the A-7D mission completion success probability (MCSP) is shown as a function of mean time between failure (MTBF) for the A-7D forward looking radar (FLR). MCSP is a measure of overall system reliability from the mission success standpoint, and in the results depicted in Figure 1, the MTBFs of all other subsystems are held constant at their operational values while the FLR MTBF is varied as shown. As shown in Figure 1, there is a wide discrepancy between the MCSP values corresponding to the operational MTBF and the MTBF requirement demonstrated by MIL-STD-781B reliability qualification testing. An exact correspondence is not to be expected between the operational MTBF and the laboratory demonstration because of differing environments and various other factors. However, one of the purposes of MIL-STD-781B testing is "facilitating the determination of more realistic correlation factors between test reliability and operational reliability." The MCSP value corresponding to the operational MTBF

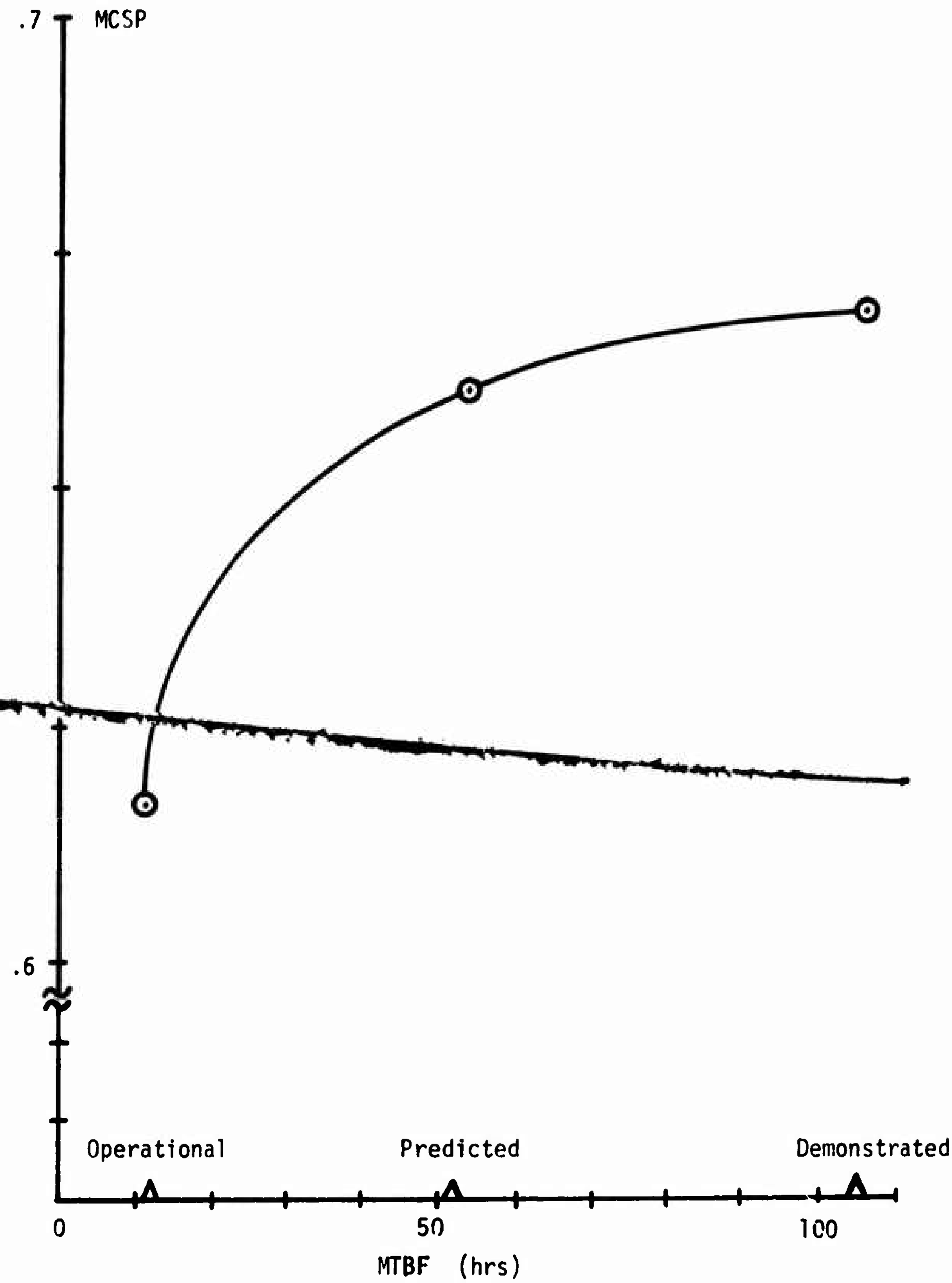


Figure 1. Mission Completion Success Probability as a Function of MTBF for the A-7D Forward Looking Radar.

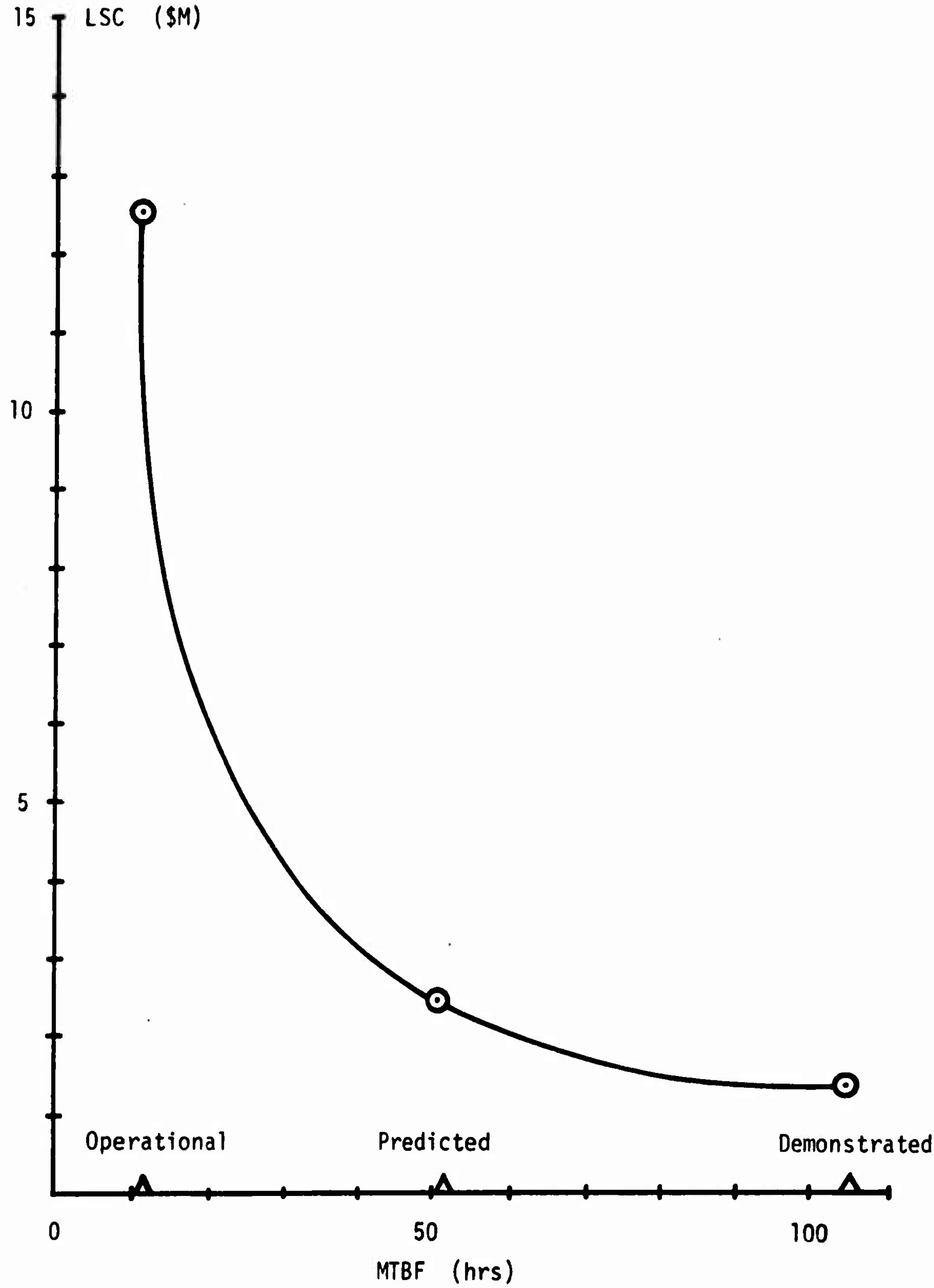


Figure 2. Logistic Support Costs as a Function of MTBF for the A-7D Forward Looking Radar.

predicted on the basis of reliability testing is also shown in Figure 1. Not only do the operational and predicted MTBFs differ significantly, but in addition, predicted system performance as measured by MCSP is not being achieved.

In Figure 2, the 10-year logistic support costs for the A-7D FLR are shown as a function of FLR MTBF. Although logistic support costs are just one part of total life cycle costs, for low reliability systems the logistic support cost can be a very significant part of total life cycle costs. Figure 2 shows the logistic support costs associated with the various MTBFs of interest. Two important points should be noted in Figure 2. Firstly, low MTBF values result in inordinate support costs, and secondly, failure to correlate reliability test results to operational levels can cause logistic support costs to be underestimated by millions of dollars.

3. OVERVIEW

In the subsequent sections, the details of the models are presented along with examples of their application. It is anticipated that these models will have a variety of uses as management tools in the systems acquisition process.

The first model to be considered is the MCSP model. An MCSP model determines the probability that the system completes its mission without experiencing an abort causing failure. (With the proper data input and interpretation of results the model can also determine the probability that the system completes its mission with degraded effectiveness, i.e., the mission is not aborted by the failure but full system capability is not available.) MCSP models are quite useful during test programs since they provide a continuous, single, easily comprehensible measure of reliability growth for the total system. They also highlight any problem areas early in the program so that appropriate action can be taken. In addition, the results of MCSP modeling techniques can provide the potential user with early insight into the operational suitability of the system from the reliability standpoint.

MCSP models by themselves are inadequate for life cycle cost analyses since they do not consider the cost of reliability development/improvement, nor do they consider logistic support costs. For example, it is of little value to determine that improving the reliability of a critical subsystem leads to dramatic enhancement of the MCSP if the cost ramifications associated with the improvement are not carefully considered. It could happen that the cost of reliability improvement is exorbitant and exceeds any expected savings in logistic support costs. On the other hand, by selecting subsystems for reliability improvement based on MCSP, the cost of reliability improvement, and logistic support cost considerations, the reliability of the total system can be improved in an optimum manner. The next step in developing the methodology is to consider reliability management, i.e., the tradeoff between reliability development/improvement costs and logistic support cost savings. Optimum reliability levels can be selected in this way.

Combining reliability optimization with MCSP considerations leads to the development of the Designing to System Performance/Cost (DSPC) model. The DSPC methodology represents a new and innovative approach to system acquisition, and preliminary results indicate that this technique will provide very valuable information to the decision-maker. This methodology systematically identifies those subsystem options which provide the highest system performance at any prescribed level of cost (either acquisition cost or acquisition plus logistic support cost). The DSPC model is compatible with designing to system cost, or performance, or both. Once total system reliability specifications are established each individual subsystem has a corresponding installed reliability and cost goal which allows realistic and continuous evaluation and adjustment as the subsystem is developed to maturity.

Two important applications of the DSPC model are in establishing reliability requirements and reliability testing. The model would determine the most realistic reliability levels for the available funding, and would also measure the cost consequences and impact on system performance if higher reliability levels are desired. When applied at the subsystem level during reliability testing, the model would determine the most cost effective technique for correcting reliability deficiencies.

Finally, a generalized approach for combining system effectiveness with the results of the DSPC methodology is presented. The input data required for this step is a valid measure of effectiveness for the system under consideration. As examples, two measures of effectiveness for fighter aircraft are developed. In the case of air-to-ground fighters, it is shown that an evaluation of the effectiveness must account for the interaction of availability, abort probability, kill potential, and survivability; and survivability is often the most dominant factor. For air-to-air fighters, the exchange ratio (Red aircraft destroyed per Blue aircraft destroyed) is an important measure of worth, and it can be expressed as a function of weapon effectiveness, maneuver capability, and first shot probability with first shot probability being the most important parameter.

The Appendix Section contains descriptions and listings for the computer programs developed in the study.

SECTION II

GENERALIZED MISSION COMPLETION SUCCESS PROBABILITY MODEL

1. INTRODUCTION

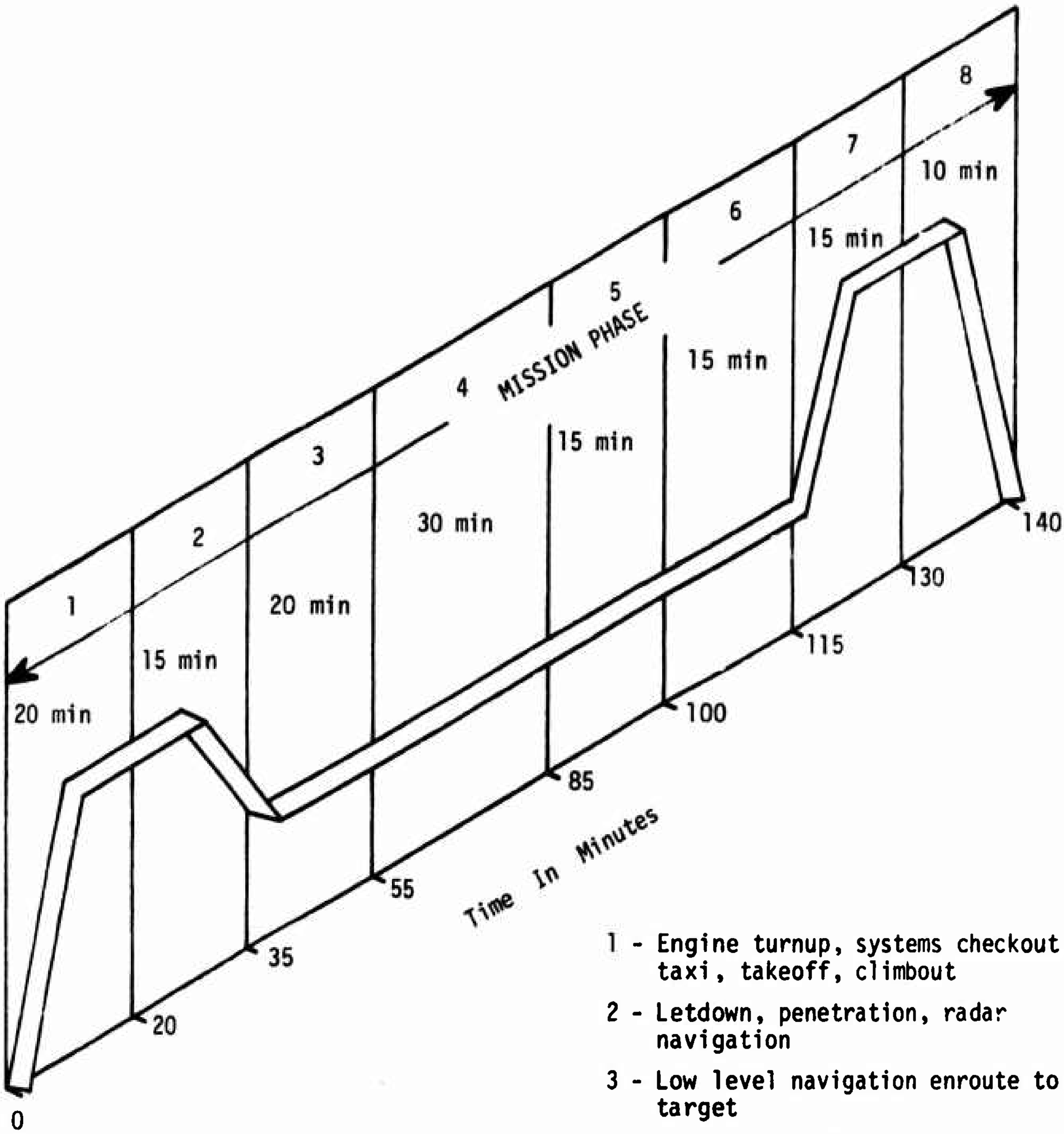
This section presents the development of a generalized MCSP model. Such a model can be applied to any system which can be divided into mission critical subsystems for which mean time between failure (MTBF) data either exists or can be estimated, and for which a mission profile can be defined. OAS experience to date is only with aircraft systems. Therefore, the examples and terminology presented in this report are aircraft oriented. A digital computer program listing for the model is presented in Appendix A.

2. MISSION PROFILE

The mission profile should be typical or representative for the given system. In addition, the profile should be divided into phases and the subsystems critical to each phase should be identified. Figure 3 is an example of a close air support mission profile for the A-7D.

3. MATHEMATICAL DEVELOPMENT

a. Basic MCSP Model. The MCSP model is based on subsystem failures which follow the exponential distribution. This distribution is characterized by a constant failure rate which is usually a valid assumption for most of the subsystems of interest. Physically, a constant failure rate indicates the subsystem has gone through a burn-in period so that failures due to design deficiencies are negligible, and also subsystem components are repaired or replaced on a regular basis so that physical wearout does not cause the failure rate to increase with time. For subsystems for which the exponential distribution is not applicable, the mathematical formulation presented below remains the same with the appropriate distribution being utilized, and the equations changed accordingly.



1 - Engine turnup, systems checkout, taxi, takeoff, climbout

2 - Letdown, penetration, radar navigation

3 - Low level navigation enroute to target

4 - Target acquisition, bomb releases, loiter in target vicinity

5 - Depart target, low level road reconnaissance, search for targets of opportunity

6 - Two gun firing passes

7 - Depart target, climbout, navigation problem

8 - Approach, landing, engine shutdown

Figure 3. A-7D Mission Profile.

It is first assumed that no subsystem is redundant. The modifications required to account for redundancy are described later. For an exponentially distributed failure pattern, the probability, P_{ic}, that the i-th subsystem completes its function without a failure is given by

$$P_{ic} = \exp\left\{ -\frac{1}{\tau_i} \sum_{j=1}^{N_p} t_{ij} \right\} , \qquad \text{(II-1)}$$

where τ_i is the mean operating time between failures for the i-th subsystem, t_{ij} is the time the i-th subsystem is used in the j-th phase, and N_p is the number of phases in the mission.

Before proceeding further with the mathematical development of the MCSP model, some discussion of failure types is warranted. This is important for the proper understanding of the very important concept of probability of abort causing failure. The importance of this concept lies in the fact that not all subsystem failures cause aborts, and furthermore, not all failures which would normally cause an abort (depending during which phase they occurred) reduce mission effectiveness to zero.

Failure types are defined according to their effect on completing the mission and the mission effectiveness. Failure types are further categorized by the mission phase during which they occur. Table I lists an example of the various failure types and their effects on mission completion and mission effectiveness.

After determing the effect on the mission of various failure types, the next step is to determine the probability that the i-th subsystem completes its function without an abort causing failure. The most convenient way to treat

Table I

FAILURE TYPES AND THEIR EFFECTS ON
MISSION COMPLETION AND MISSION EFFECTIVENESS

	TYPE	EFFECT ON MISSION	MISSION EFFECTIVENESS
Before Mission Objective Phase	Reduction in Safety	Aborted	None
	Extreme Reduction in Capability	Aborted	None
	Reduction in Capability	Completed	Reduced
	Minor Malfunctions	Completed	Full Capability
During Mission Objective Phase	Reduction in Safety	Aborted	Reduced
	Extreme Reduction in Capability	Completed	Reduced
	Reduction in Capability	Completed	Reduced
	Minor Malfunctions	Completed	Full Capability
After Mission Objective Phase	Reduction in Safety	Completed	Full Capability
	Extreme Reduction in Capability	Completed	Full Capability
	Reduction in Capability	Completed	Full Capability
	Minor Malfunctions	Completed	Full Capability

this is to introduce the concept of mean operating time between abort causing failures. For this discussion let τ denote the mean operating time between failures for a certain subsystem. This means that the various failure modes have been defined for that subsystem. Suppose there are n different failure modes possible for the subsystem, i.e., in the determination of τ each failure had to be classified as one of these n modes. Let the n failure modes be denoted by

$$f_1, f_2, \ldots, f_n \quad . \tag{II-2}$$

Suppose the first k failure modes are abort causing failures. Given that a failure occurs, the relative frequency of occurrence of the first k failure modes is some number P_a. This value P_a is the probability of abort given a failure of the subsystem. If the subsystem operates for time T then the expected number of failures is

$$\frac{T}{\tau} \quad . \tag{II-3}$$

Since P_a is the fraction of failures causing an abort, the expected number of abort causing failures is

$$P_a \frac{T}{\tau} \quad . \tag{II-4}$$

Letting τ_a denote the mean operating time between abort causing failures, then it follows from the definition of τ_a that the expected number of abort causing failures is given by

$$\frac{T}{\tau_a} = P_a \frac{T}{\tau} \quad . \tag{II-5}$$

From equation (II-5) the relation between τ_a and τ is found to be

$$\tau_a = \frac{\tau}{P_a} . \qquad \text{(II-6)}$$

Clearly, the value of P_a is dependent upon the definition of failure used in the determination of τ. It also depends on how critical the subsystem is during the j-th phase.

To clarify these ideas, suppose a subsystem has 6 failure modes where the consequence and effect on the mission for each failure mode is given in Table II.

Table II

FAILURE MODES AND THEIR EFFECT ON MISSION

FAILURE MODES	CONSEQUENCE	EFFECT ON MISSION
f_1	Extreme Reduction in Effectiveness	Abort
f_2	Reduction in Safety	Abort
f_3	Reduction in Safety	Abort
f_4	Reduced Effectiveness	Continue with Reduced Eff.
f_5	Reduced Effectiveness	Continue with Reduced Eff.
f_6	Minor Repairs Required	None

Since only failure modes f_1, f_2, and f_3 cause an abort, it follows that

$$P_a = \frac{P\left\{f_1, f_2, f_3\right\}}{P\left\{\text{failure}\right\}} , \qquad \text{(II-7a)}$$

where $P\{f_1, f_2, f_3\}$ denotes the probability that failure mode f_1 or f_2 or f_3 occurs. The probability of abort due to safety factors (given that the subsystem fails) is

$$P_{as} = \frac{P\{f_2, f_3\}}{P\{\text{failure}\}} \quad . \tag{II-7b}$$

The probability of reduced (or zero) effectiveness (given a failure) is

$$P_E = \frac{P\{f_1, f_2, f_3, f_4, f_5\}}{P\{\text{failure}\}} \quad . \tag{II-7c}$$

The point to be made is that such factors as P_{as} and P_E can be used in the same manner as P_a to calculate other measures, for instance, the probability of completing the mission without a safety abort or the probability of completing the mission with maximum effectiveness.

The failure modes, the associated failure rates, and the impact on mission performance can be estimated for a new subsystem design by component analysis, initial testing, or from Air Force Logistics Command data for similar systems. As the development of the subsystem progresses these estimates can be updated.

Using the concept of mean operating time between abort causing failures, the probability that the i-th subsystem does not cause an abort (given that the mission was not aborted due to other causes) is given by

$$\begin{aligned} P_i &= \prod_{j=1}^{N_p} \exp\left(-\frac{t_{ij}P_{aij}}{\tau_i}\right) \\ &= \exp\left(-\frac{1}{\tau_i}\sum_{j=1}^{N_p} t_{ij}P_{aij}\right) \quad , \end{aligned} \tag{II-8}$$

where P_{aij} is the probability of mission abort given that the i-th subsystem fails during the j-th phase. The abort probability P_{aij} depends upon how mission critical the i-th subsystem is during the j-th phase. Since it is the relative frequency of failures which are abort causing failures (abort type failures), P_{aij} is also dependent upon the definition of a failure. In most cases a failure is considered an abort type failure for reasons of safety or reduced effectiveness.

By calculating P_i for each subsystem, the subsystems can be ranked according to their likelihood of aborting the mission. An example of this aspect of the MCSP model is presented below.

The next item of interest is the probability $P_{c\ell}$ of completing the ℓ-th phase without an abort causing failure. (In order to reach the ℓ-th phase all previous phases must have been completed without an abort causing failure.) This probability is given by

$$P_{c\ell} = \prod_{i=1}^{N_s} \exp\left\{ -\frac{1}{\tau_i} \sum_{j=1}^{\ell} t_{ij} P_{aij} \right\} , \qquad \text{(II-9)}$$

where N_s is the total number of subsystems and ℓ is the mission phase of interest. The case $\ell = N_p$ yields the mission completion success probability

$$MCSP = \prod_{i=1}^{N_s} P_i , \qquad \text{(II-10)}$$

where P_i is given by equation (II-8).

MCSP by cumulative phases is of interest because it makes it possible to examine the mission up to and including any phase. For example, in the case of a single mission, abort causing failures occurring after the target phase do not affect mission effectiveness. However, in the more interesting cases involving repeated sorties, failures occurring during all phases are important since they affect maintenance requirements between sorties. An important measure of maintenance requirements is the probability of completing the mission without any subsystem failures. This measure is obtained by setting all abort probabilities, P_{aij}, equal to unity and using equations (II-8) and (II-10).

Two other items of interest regarding MCSP are the probability, P_{ij}, that the i-th subsystem causes an abort in phase j given no abort before phase j; and the probability, P_{apj}, of abort in phase j given no abort before phase j. These probabilities are given respectively by

$$P_{ij} = 1 - \exp\left(- \frac{t_{ij}P_{aij}}{\tau_i} \right) ,$$

and (II-11)

$$P_{apj} = 1 - \exp\left\{ - \sum_{i=1}^{N_s} \frac{t_{ij}P_{aij}}{\tau_i} \right\} .$$

Examples of applying the methodology are presented in Table III and Figure 4. Table III shows the critical subsystem identification for the A-7D. On the left hand side, the eight A-7D subsystems with the highest failure rates during Category II testing are shown. On the right hand side, the ranking of the eight A-7D subsystems causing the greatest number of aborts during Category II testing are shown. (The number in parenthesis, $1 - P_i$, is the probability that the subsystem will cause an abort during the mission.) An examination of the Table reveals that only the Forward Looking Radar (FLR) preserves the same ranking. Some subsystems appearing in the MTBF ranking do not appear in the abort ranking and vice versa. This illustrates the fact that MTBF alone is not a good indicator of the effect a subsystem will have on mission success.

Table III

CRITICAL SUBSYSTEM IDENTIFICATION

MTBF RANKING			ABORT RANKING		
		MTBF			$1 - P_i$
1.	Forward Looking Radar	(12 hr)	1.	Forward Looking Radar	(.117)
2.	Inertial Measurement System	(31 hr)	2.	Navigation Weapon Delivery Computer	(.064)
3.	Lighting	(34 hr)	3.	Inertial Measurement System	(.056)
4.	Navigation Weapon Delivery Computer	(35 hr)	4.	M61 Gun	(.047)
5.	M61 Gun	(38 hr)	5.	Tactical Air Navigation	(.032)
6.	Tactical Air Navigation	(44 hr)	6.	Radar Altimeter	(.032)
7.	Radar Altimeter	(44 hr)	7.	Head Up Display	(.030)
8.	Landing Gear	(64 hr)	8.	Weapons Release	(.027)

The abort ranking is dependent upon the length of time the subsystem is used during the mission, the MTBF of the subsystem, and the conditional probability that the mission will be aborted given that the subsystem fails. Thus, in general, the abort ranking does not correspond to the MTBF ranking. This example illustrates the way the MCSP model can be utilized to identify those subsystems whose reliability improvement most enhances probability of mission completion. The next example illustrates the evaluation of those subsystems so identified.

Figure 4 shows the results of the type of sensitivity analysis that can be conducted using the MCSP model. Starting with the baseline system, the effect of improving the reliability of single subsystems or combination of subsystems can be analyzed. The abort ranking in the previous Table identified the Forward Looking Radar (FLR) and the Navigation Weapon Delivery Computer (NWDC) as the two A-7D subsystems having the most impact on mission success. Increasing the MTBF's of the NWDC and the FLR results in dramatic improvements in MCSP, while increasing the MTBF of relatively high reliability subsystems such as the engine has essentially no effect on MCSP. However, it does not follow that the reliability of the engine should not be improved since it is possible that the cost of improvement could be more than compensated for by the resultant savings in logistic support cost. These important considerations will be discussed later.

The methodology presented so far can be used to analyze a large number of systems. In the sections below, extensions of the basic methodology which may be of interest in other applications are presented.

b. Redundant Subsystems. To achieve an increase in system reliability it may be necessary to introduce redundant subsystems provided, of course, certain constraints such as weight and volume can be met. If a subsystem has redundant units then in the expression (II-10) for MCSP, the probability that the subsystem's function is performed successfully must be adjusted to account for redundancy. The purpose of this section is to derive the expressions for the successful performance of a redundant subsystem's function and also the associated logistic support cost resulting from redundancy. Two types of

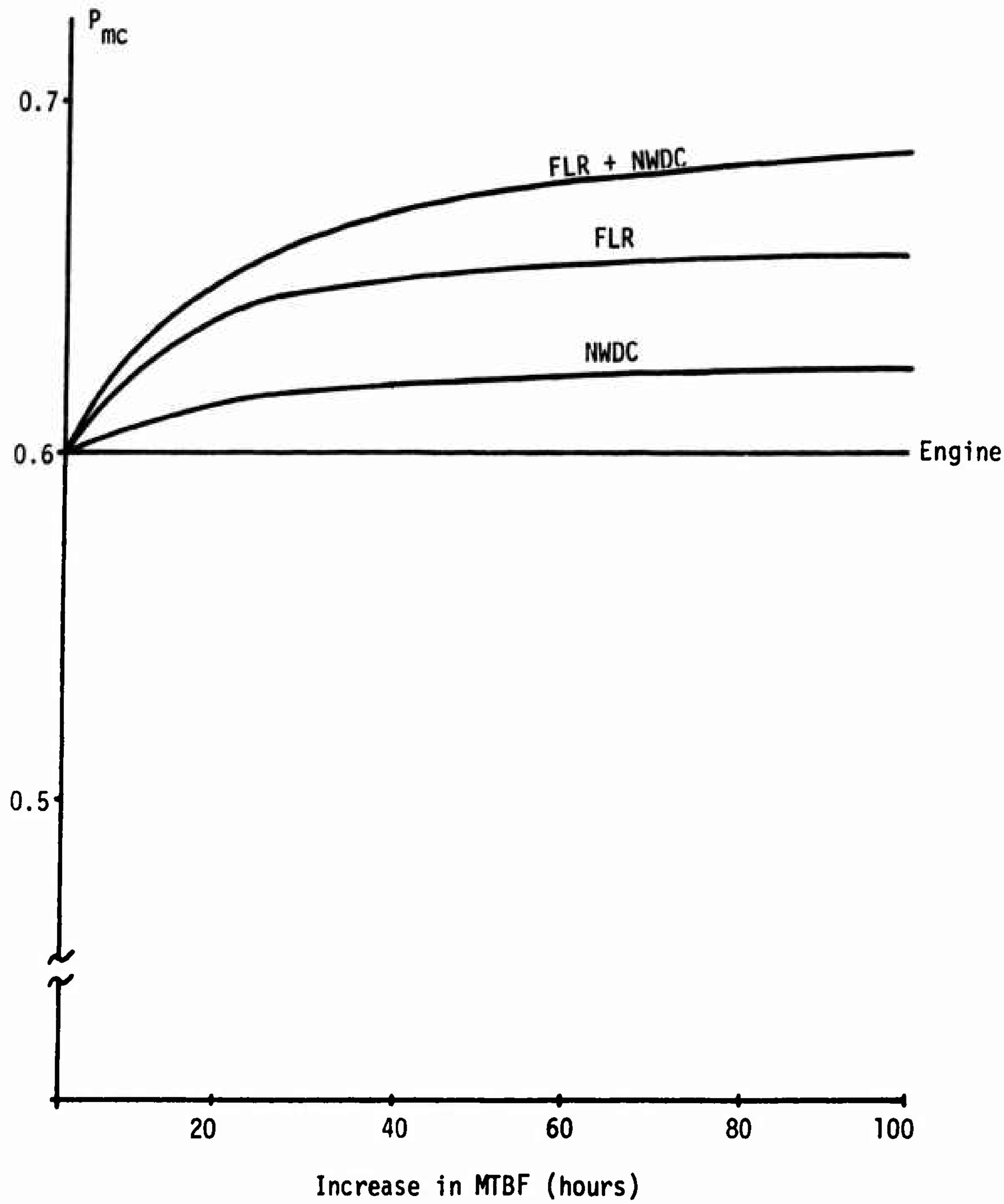


Figure 4. Evaluation of Critical Subsystem Improvement.

redundancy will be considered. The first type is Operative Redundancy which will here mean that all redundant units operate simultaneously and an abort occurs only if all units have an abort causing failure. The second type of redundancy is Standby Redundancy meaning that only one unit is operating and a redundant unit will be switched on only when the operating unit has an abort causing failure. It is assumed that there are no switching failures.

Let n denote the total number of redundant units for a subsystem, τ_{ri} the MTBF of the i-th redundant unit, and p_{ai} the probability that a failure of the i-th redundant unit is an abort causing type.

Operative Redundancy will be considered first. If the subsystem function must be performed for time T during a successful mission then the probability that the subsystem will not cause an abort is

$$P_c(n, T) = 1 - \prod_{i=1}^{n} \left\{ 1 - \exp\left(- \frac{Tp_{ai}}{\tau_{ri}} \right) \right\} . \quad \text{(II-12)}$$

If all redundant units have the same characteristics (i.e., $p_{ai} = p_a$, $\tau_{ri} = \tau_r$, i = 1, 2, ..., n) then (II-12) becomes

$$P_c(n, T) = 1 - \left\{ 1 - \exp\left(- \frac{Tp_a}{\tau_r} \right) \right\}^n . \quad \text{(II-13)}$$

In implementing the DSPC model described in Section IV it is important to know the total cost (acquisition plus logistic support costs) of n redundant units. If all units are the same then the acquisition cost for n-th order redundancy is

$$nC_a \quad , \quad \text{(II-14)}$$

where C_a is the unit acquisition cost. Since each unit operates the same amount of time the logistic support cost for Operative Redundancy is

$$nC_s \quad , \tag{II-15}$$

where C_s is the logistic support cost for a single unit. Thus, increasing the reliability by going from a single subsystem to n units with Operative Redundancy increases total cost by a factor of n. It will be shown that this is not true for Standby Redundancy.

It is now assumed that a subsystem is Standby Redundant and that all units have the same characteristics (i.e., each unit characterized by p_a and τ_r). Let τ_{ar} denote the mean time between abort type failures, i.e.,

$$\tau_{ar} = \frac{\tau_r}{p_a} \quad . \tag{II-16}$$

With Standby Redundancy only one unit is operating and a redundant unit is switched on only when the operating unit has an abort type failure. The probability that the subsystem will cause an abort will first be determined for the case of two units (primary and backup unit).

The probability that the primary unit fails during the small time interval $(t, t + \Delta t)$ is the product of the probability that the unit operates successfully for time t multiplied by the probability it fails during the next time interval Δt, i.e.,

$$\left\{ \exp\left(- \frac{t}{\tau_{ar}} \right) \right\} \frac{\Delta t}{\tau_{ar}} \quad . \tag{II-17}$$

If the primary unit fails at time t the backup unit is switched on to operate the remaining time T-t. The probability that the backup unit fails during time T-t is

$$1 - \exp\left(- \frac{T - t}{\tau_{ar}}\right) \quad . \tag{II-18}$$

Therefore, the probability that both units fail is obtained by summing (integrating) the product of (II-17) and (II-18) over all possible failure times of the primary unit. Thus,

$$1 - P_c(2, T) = \int_0^T \exp(- \frac{t}{\tau_{ar}})\left\{1 - \exp(- \frac{T - t}{\tau_{ar}})\right\} \frac{dt}{\tau_{ar}} \quad . \tag{II-19}$$

Integration of (II-19) yields

$$P_c(2, T) = \exp(- \frac{T}{\tau_{ar}})(1 + \frac{T}{\tau_{ar}}) \quad . \tag{II-20}$$

In general

$$1 - P_c(j, T) = \int_0^T \exp(- \frac{t}{\tau_{ar}})\left\{1 - P_c(j - 1, T - t)\right\} \frac{dt}{\tau_{ar}} \quad . \tag{II-21}$$

Repeated application of (II-21) yields

$$P_c(n, T) = \exp(- \frac{T}{\tau_{ar}}) \sum_{i = 0}^{n - 1} \frac{1}{i!}\left(\frac{T}{\tau_{ar}}\right)^i \quad . \tag{II-22}$$

In other words, $P_c(n, T)$ is equal to $P_c(1, T)$ multiplied by the first n terms in the expansion of $\exp\left(\frac{T}{\tau_{ar}}\right)$. Observe,

$$\lim_{n \to \infty} P_c(n, T) = \exp\left(- \frac{T}{\tau_{ar}}\right)\exp\left(\frac{T}{\tau_{ar}}\right) = 1. \tag{II-23}$$

Probably the most common redundancy is with two units, i.e., n = 2. To calculate the logistic support cost of the backup unit it is necessary to know its operating time. During a successful mission the subsystem operates for time T; the average operating time of the second unit is

$$\int_0^T (T - t)\exp\left(- \frac{t}{\tau_{ar}}\right)\frac{dt}{\tau_{ar}}$$

$$= T\left[1 - \exp\left(- \frac{T}{\tau_{ar}}\right)\right] + \tau_{ar}\left(1 + \frac{T}{\tau_{ar}}\right)\exp\left(- \frac{T}{\tau_{ar}}\right) - \tau_{ar}$$

$$= T - \tau_{ar}\left[1 - \exp\left(- \frac{T}{\tau_{ar}}\right)\right] \approx \frac{T^2}{2\tau_{ar}} \quad . \tag{II-24}$$

Dividing by T, the average fraction of the total time T the second unit is operating is then

$$\frac{T}{2\tau_{ar}} \quad . \tag{II-25}$$

Thus, the logistic support cost for the second unit can be approximated by

$$\frac{T}{2\tau_{ar}} C_s \quad . \tag{II-26}$$

This means that the change from one unit to two units increases the subsystem reliability from

$$P_c(1, T) = \exp\left(- \frac{T}{\tau_{ar}}\right) \tag{II-27a}$$

to

$$P_c(2, T) = \exp\left(-\frac{T}{\tau_{ar}}\right)\left(1 + \frac{T}{\tau_{ar}}\right) \quad . \tag{II-27b}$$

Furthermore, the corresponding increase in total unit cost is

$$\Delta C = C_a + \frac{T}{2\tau_{ar}} C_s \quad . \tag{II-28}$$

The increase in cost for Standby Redundancy can be considerably less than that with Operative Redundancy. In Section IV some of the above redundancy equations will be utilized in an example to show how the DSPC model will identify conditions under which redundancy is the optimal choice.

Next to be considered is the case of two units with Standby Redundancy where the MTBF's of the primary and backup units are different. Letting τ_{a1} and τ_{a2} denote the mean system operating time between abort causing failures of the primary and backup units, respectively, the expression for $P_c(2, T)$ becomes

$$P_c(2, T) = 1 - \int_0^T \exp\left(-\frac{t}{\tau_{a1}}\right)\left\{1 - \exp\left(-\frac{T-t}{\tau_{a2}}\right)\right\}\frac{dt}{\tau_{a1}}$$

$$= \exp\left(-\frac{T}{\tau_{a1}}\right)\left(\frac{\tau_{a1}}{\tau_{a1} - \tau_{a2}}\right) + \exp\left(-\frac{T}{\tau_{a2}}\right)\left(\frac{\tau_{a2}}{\tau_{a2} - \tau_{a1}}\right) \quad . \tag{II-29}$$

For n units with Standby Redundancy, all MTBF's being different, it can be shown that

$$P_c(n, T) = \sum_{i=1}^{n} \left\{ \frac{\tau_{ai}^{\,n-1} \exp\left(-\frac{T}{\tau_{ai}}\right)}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{n} (\tau_{ai} - \tau_{aj})} \right\} \quad , \tag{II-30}$$

where τ_{ai} denotes the mean operating time between an abort causing failure of the i-th redundant subsystem.

It is instructive to illuminate some of the above ideas by means of a s.mple example of a subsystem which operates 3 hours during a mission, $p_a = 0.8$, $\tau_r = 10$ hrs, $C_a = \$1M$, and $C_s = \$3M$. Table IV shows the increase in performance and cost resulting from redundancy. It also shows the advantage of Standby Redundancy, i.e., Operative Redundancy is equivalent to increasing the subsystem MTBF from 10 hours to 55 hours with total cost increasing from \$4M to \$8M, whereas Standby Redundancy is equivalent to an MTBF of 96 hours at a cost of \$5.4M.

Table IV

REDUNDANCY EXAMPLE

NUMBER OF UNITS	TYPE REDUNDANCY	MCSP	MTBF EQUIVALENT (hrs)	ACQ. COST (\$M)	15-YEAR LSC (\$M)	TOTAL COST (\$M)
1	None	.7927	10	1	3	4
2	Operative	.9570	.55	2	6	8
2	Standby	.9754	96	2	3.4	5.4

4. SUMMARY

In this section the basic MCSP methodology has been presented along with clarifying examples and extensions of the basic model to include redundant subsystems.

The MCSP model can be used to assess the reliability of the total system based on the reliability of the individual subsystems; rank the subsystems in terms of the probability of abort causing failures; and determine the MCSP enhancement due to improvements in individual subsystem reliability.

In the next section reliability management techniques are discussed. These techniques include the cost considerations that must be combined with the MCSP results in order to extend the methodology for applications to life cycle cost analyses.

SECTION III

RELIABILITY MANAGEMENT

1. INTRODUCTION

MCSP models are quite useful for identifying critical subsystems and also for determining the enhanced performance to be gained by improving the reliability of the critical subsystems. However, as mentioned previously, MCSP models by themselves are inadequate for life cycle cost analyses since they do not take into account the cost of reliability development/improvement or logistic support costs. For example, Figure 4 in the previous section showed that improving the MTBF of the engine had essentially no effect on MCSP. However, improvements in engine MTBF could significantly decrease logistic support costs. On the other hand, reliability improvement of the Navigation Weapon Delivery Computer and the Forward Looking Radar significantly improves MCSP, but the cost might be so prohibitive as to preclude reliability improvement for these subsystems.

2. METHODOLOGY

a. Logistic Support Cost. Logistic support costs can be conveniently analyzed by considering the average cost per repair on a subsystem basis. The average cost per repair for a given subsystem is determined by dividing the yearly logistic support cost by the number of subsystem failures during the yearly period. This data is compiled by the AFLC Air Materiel Areas and includes Field Maintenance Cost, Specialized Repair Activity Cost, Packing and Shipping Cost, Condemnation Cost, and Base Material Cost. Once the average cost per repair is established in this way, logistic support cost projections can be made for future years as shown below. (For subystems not in the inventory, estimates must be made based on similar subsystems of comparable complexity.)

The average yearly logistic support cost for the i-th subsystem, LSC_i, is given by

$$LSC_i = (CR_i)(R_i) \quad , \qquad \text{(III-1)}$$

where CR_i is the average cost per repair for the i-th subsystem, and R_i is the expected number of repairs for the i-th subsystem during the year. The expected number of repairs can be expressed as

$$R_i = \frac{T_i}{\tau_i} \quad , \qquad \text{(III-2)}$$

where T_i is the yearly operating time of the i-th subsystem, and τ_i is the MTBF of the i-th subsystem.

Therefore, the average yearly logistic support cost for the i-th subsystem is given by

$$LSC_i = (CR_i)\left(\frac{T_i}{\tau_i}\right) \quad , \qquad \text{(III-3)}$$

and the logistic support cost for y years is given by

$$LSC_{iy} = (y)(CR_i)\left(\frac{T_i}{\tau_i}\right) \quad . \qquad \text{(III-4)}$$

The total system logistic support cost is given by

$$LSC_y = y \sum_{i=1}^{N_s} (CR_i)\left(\frac{T_i}{\tau_i}\right) \quad , \qquad \text{(III-5)}$$

where N_s is the total number of subsystems.

As an example, Table V shows MTBF and cost per repair data for three hypothetical subsystems.

Table V

HYPOTHETICAL SUBSYSTEM MTBF AND AVERAGE COST PER REPAIR DATA

SUBSYSTEM	MTBF (hours)	AVERAGE COST PER REPAIR ($)
A	15	100
B	300	200
C	475	1,000

Assuming that the total 10-year operating time for the three subsystems is 3×10^6 hours (fleet size = 500, average monthly operating time = 50 hours), the relationship between 10-year logistic support cost, MTBF, and average cost per repair can be presented as shown in Figure 5. (Any other value of operating time would result in merely a change of scale for the ordinate and would not change the conclusions.) In Figure 5, it is immediately apparent that:

(1) Subsystem A needs MTBF improvement. (Even a small increase in MTBF will result in significant logistic support cost savings.)

(2) Subsystem B appears satisfactory (relative to the other subsystems).

(3) With Subsystem C a reduction in the cost per repair (rather than MTBF improvement) could lead to significant savings.

Figure 5 shows the ramifications of reliability and cost per repair on logistic support costs. Although it is obvious that the way to reduce logistic support cost is to improve reliability and/or reduce the cost of repair; when this type of analysis is applied to each subsystem it systematically establishes priorities, indicates realistic goals, and allows for the proper allocation of resources.

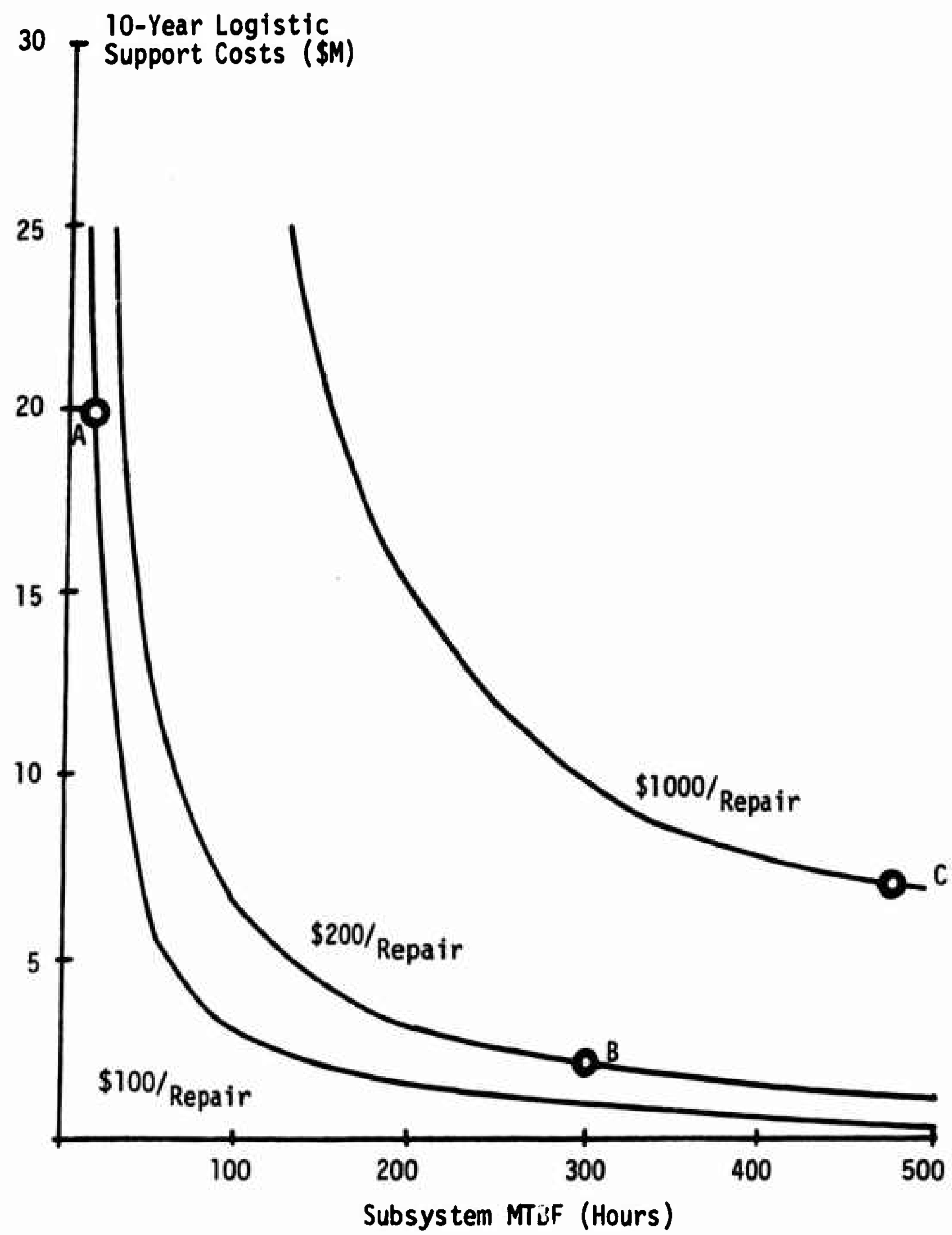


Figure 5. Ten-Year Logistic Support Cost for Three Hypothetical Subsystems.

b. Reliability Optimization. Reliability optimization, i.e., the trade-off between levels of reliability and lifetime support cost to decrease system life cycle cost is a very important adjunct to MCSP considerations. Reliability levels can be established during development by analyzing subsystem options and their associated costs, or by reliability improvement programs applied after the system becomes operational.

A hypothetical example of reliability optimization is shown in Figure 6. In Figure 6.a, logistic support costs for a given subsystem are seen to decrease as a function of MTBF as shown in Figure 5. Figure 6.b depicts the development or improvement costs associated with establishing various levels of subsystem reliability. The life cycle cost of a subsystem is the sum of the acquisition costs, logistic support costs, and operating costs. A cost reduction in any of these areas leads to reduced life cycle costs. Reliability levels can be established either during subsystem development or through reliability improvement programs such that the sum of lifetime logistic support costs and reliability development/improvement costs can be minimized. This is shown in Figure 6.c where lifetime logistic support costs and reliability development/improvement costs are combined as a function of subsystem MTBF. Thus, reliability goals can be selected which minimize the sum of reliability improvement plus logistic support costs. This same procedure can be utilized in conjunction with MCSP models to reduce acquisition costs. This subject is addressed in Section 2-d.

c. Cost of Repair. Another approach to logistic support cost reduction is through reducing the repair costs for certain subsystems. Specific methodology cannot be developed for systematically reducing repair costs, and the problem must be dealt with on a subsystem by subsystem basis. In general, during design and development repair considerations should be

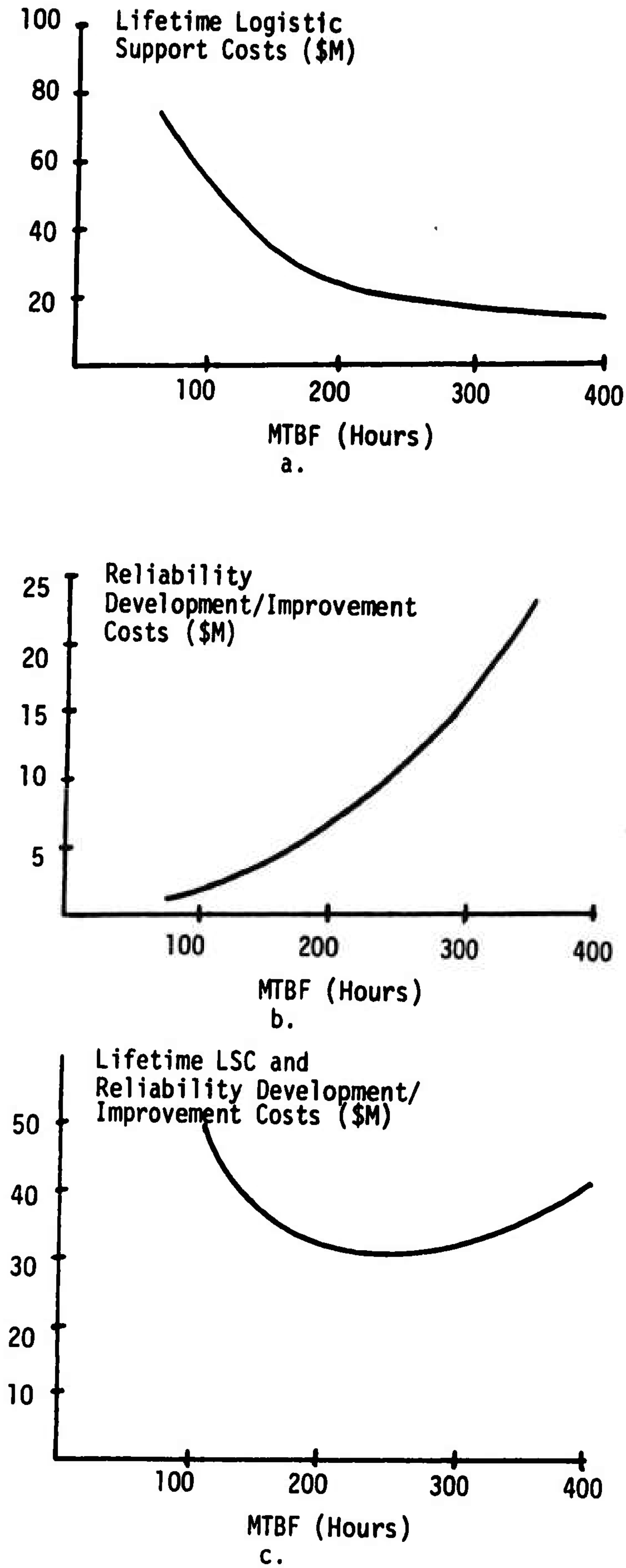


Figure 6. Reliability Optimization.

emphasized which make equipment easily accessible for inspection or removal. Also designs utilizing standardized components, tools, and test equipment can significantly reduce maintenance costs. During the B-1 Mockup Review, 297 Requests for Alteration (RFA) were developed and processed. Of the 93 RFA involving maintainability, 34 focused on accessibility. Although precise estimates of the potential savings from this type of preliminary maintainability assessment are not available, it is clear that such emphasis on minimizing repair costs during the early stages of a program can have a significant impact on lifetime logistic support costs.

After a subsystem becomes operational and experiences very high repair costs, corrective action can sometimes be taken through Increased Reliability of Operational Systems (IROS) programs. IROS programs attempt to pinpoint causes of low reliability or high repair costs and then make recommendations for modifying the equipment to alleviate these problems. An excellent example of this can be drawn from the IROS program on the A-7D Air Data Computer.

The A-7D Air Data Computer has experienced excessive logistic support costs due to a water ingestion problem associated with the pitot static system. Generally, when water gets into the Air Data Computer it must be returned to the depot for overhaul, and this is the major contributor to the high logistic support cost. A modification program is currently under way to correct this problem. Figure 7 shows the estimated savings in logistic support costs that can be expected after the A-7D fleet is modified. By solving the water ingestion problem, the average cost per repair for the Air Data Computer will be decreased significantly. As shown in Figure 7, the reduction in average cost per repair is dramatically more cost effective than doubling the MTBF of the unmodified subsystem. Along with the cost per repair

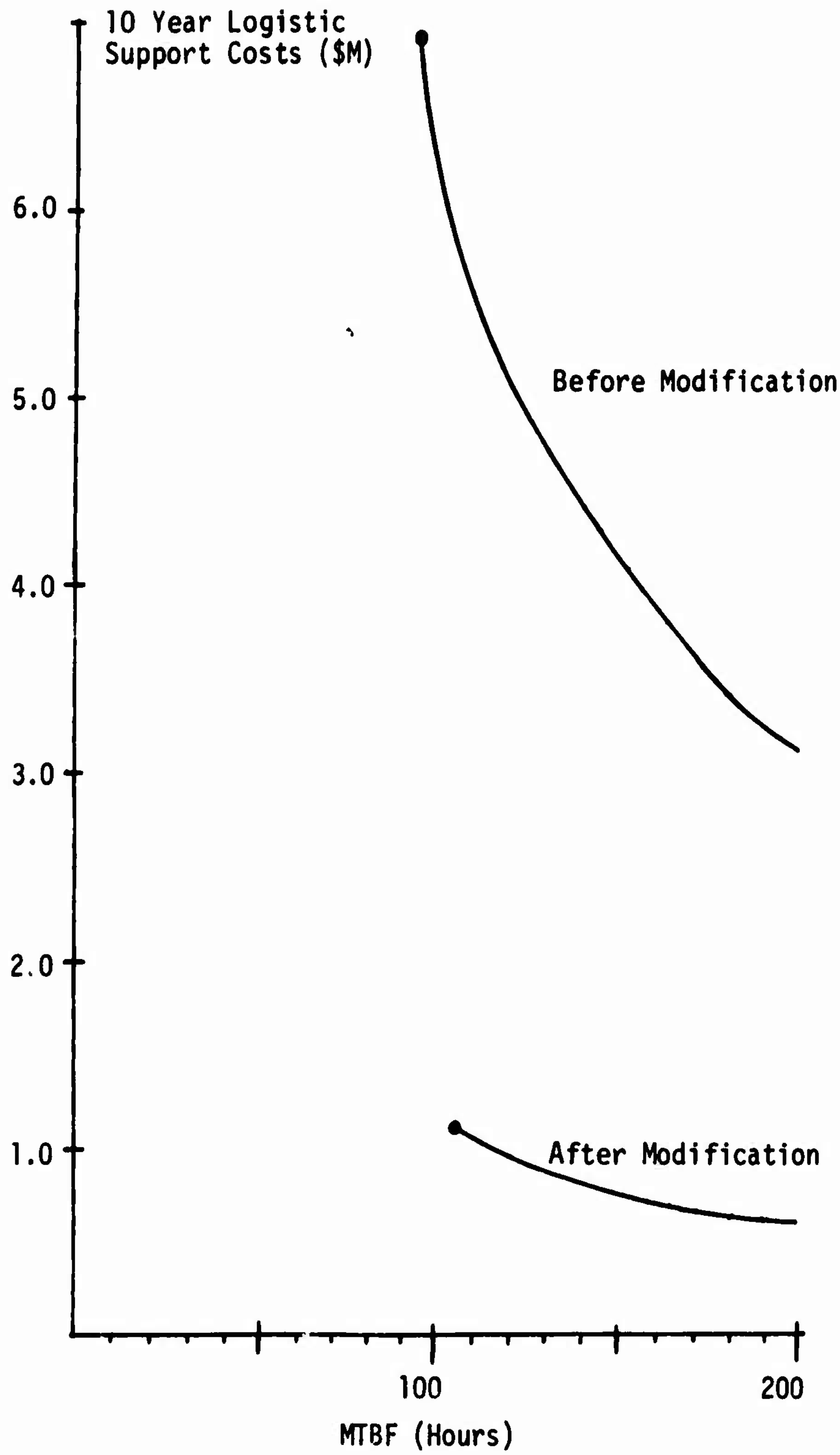


Figure 7. Ten-Year Logistic Support Costs as a Function of MTBF for the A-7D Air Data Computer.

reduction a 10-20 percent increase in MTBF is expected after modification.

d. MCSP and Logistic Support Costs. The previous section demonstrated how logistic support cost savings can be made after a system becomes operational. By utilizing MCSP models and analyzing logistic support costs, savings can also be effected during the acquisition phase of system procurement. Section 3.b demonstrated that logistic support cost can be minimized if the optimum subsystem MTBF can be realized. However, it is not always possible to design to optimum levels of reliability because of various constraints such as time factors, limited funding, and technological barriers. Therefore, additional guidance is required in order to establish realistic reliability goals for each subsystem. MCSP models provide this guidance.

In order to obtain required system performance for the least costs, there should be several options available for each candidate subsystem. Figure 8 shows a hypothetical example of subsystem reliability options. In Section 3.b the reliability development/improvement graph was shown as a continuous curve. Actually such graphs would consist of discrete points since reliability levels would be established in discrete steps rather than continuously. Figure 8 shows three options which may represent the same subsystem modified in two cases and an entirely different subsystem performing the same function in the third case, or any combination thereof. The length of the lines for each option represent the lower and upper limits or ranges of the expected MTBF of the subsystem. With subsystem reliability options available, MCSP models and logistic support cost data can be used to select the most appropriate option for each subsystem. This selection will not necessarily be the optimum as shown in Figure 6.c. For example, it may not be possible to achieve the optimum MTBF for a given subsystem because of the constraints associated with reliability development/improvement mentioned

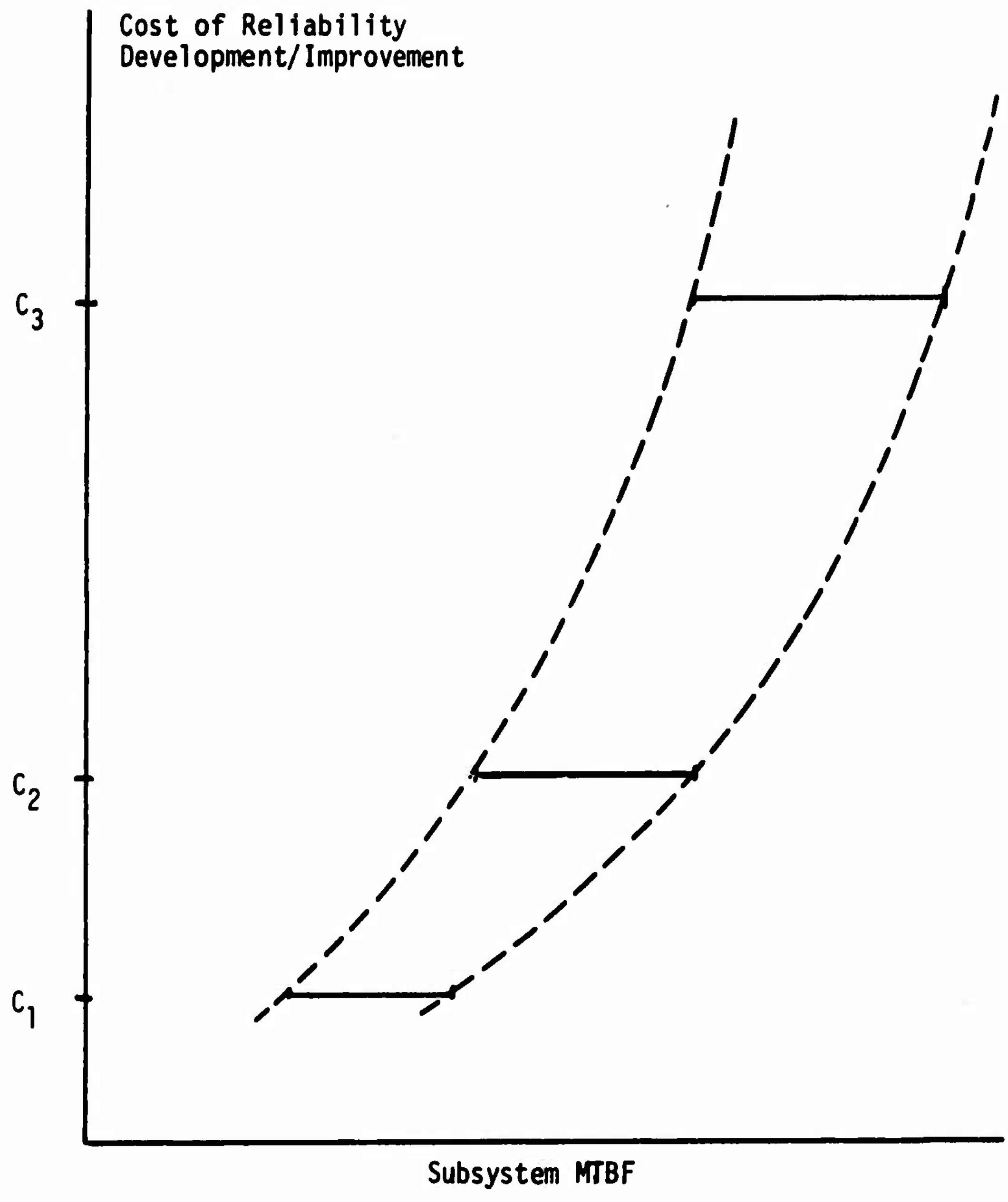


Figure 8. Options for Performance/Cost Determinations.

previously. Also, if the subsystem is especially critical to successful completion of the mission, it may be necessary to spend money nonoptimally (right hand side of curve in Figure 6.c) for some subsystems in order to achieve the required performance (MCSP) while not improving the reliability of other less critical subsystems at all. The net result is the achievement of a given level of performance for the least cost.

The importance of proper reliability management cannot be overemphasized. Figure 9 shows the consequences of not considering required system performance and logistic support costs in establishing reliability goals. Figure 9.a shows the mission completion success probability as a function of MTBF for the A-7D Navigation Weapon Delivery Computer (NWDC). All other subsystem MTBFs have been held constant at Category II values. The tick mark at the left indicates the MTBF of the NWDC achieved during Category II testing (35 hours). This value has been improved somewhat since the A-7D has become operational, but it is still well below the mature system predicted level indicated by the second tick mark (499 hours). However, an examination of the curve shows that as far as probability of mission completion (P_{mc}) is concerned there is no reason to improve the MTBF beyond about 150 hours. The only other reason for high MTBF requirements would be to reduce logistic support costs. Figure 9.b shows the logistic support cost (LSC) as a function of MTBF for the NWDC where tick marks are again used to indicate the Category II and mature system MTBFs. As shown on the curve, a point of diminishing returns in LSC savings is reached for MTBFs greater than about 200 hours. Since Category II, the mature system MTBF prediction for the NWDC has been revised to 250 hours. This is a much more realistic value. Unfortunately, reliability development/improvement data is not available for the NWDC. Such data would complete the analysis of the NWDC from the reliability management standpoint. Even without the reliability

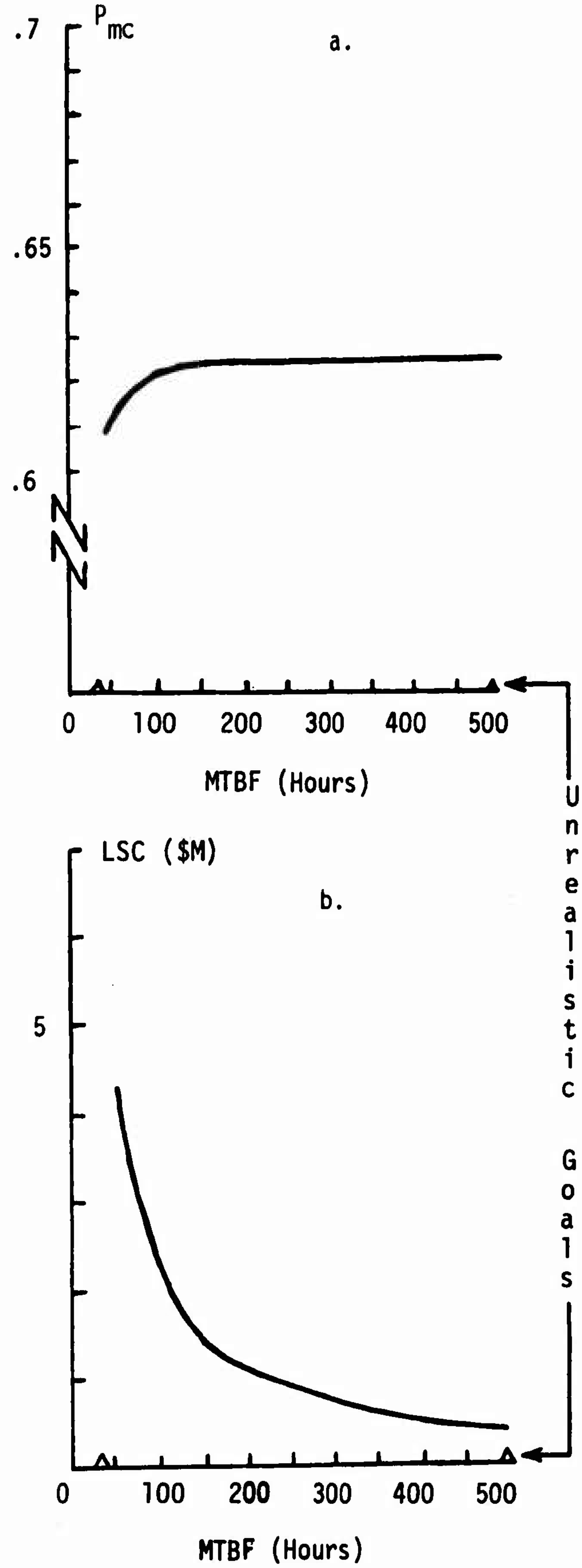


Figure 9. P_{mc} and Logistic Support Cost as a Function of MTB F for the A-7D Navigation Weapon Delivery Computer.

development/improvement data this example demonstrates the first principle of optimization in system development, i.e., don't buy or strive for reliability levels that are unreasonable or unrealistic.

3. SUMMARY

This section has described a procedure that ensures obtaining required system performance levels for minimum costs. This methodology is depicted graphically in Figure 10. If the information displayed in Figure 10 is available for most of the mission critical subsystems (generally options will not be available for every subsystem), realistic goals can be established and options can be selected such that the required performance of the overall system is obtained for minimum cost. The major limitation in this approach is that curves such as those displayed in Figure 10 must be examined for each subsystem for which they are available, and it is difficult and cumbersome to establish priorities. This is particularly critical if funds are limited. The next section discusses a procedure that systematically and in a step by step fashion selects the options that offer the biggest payoffs in terms of higher performance/lower costs.

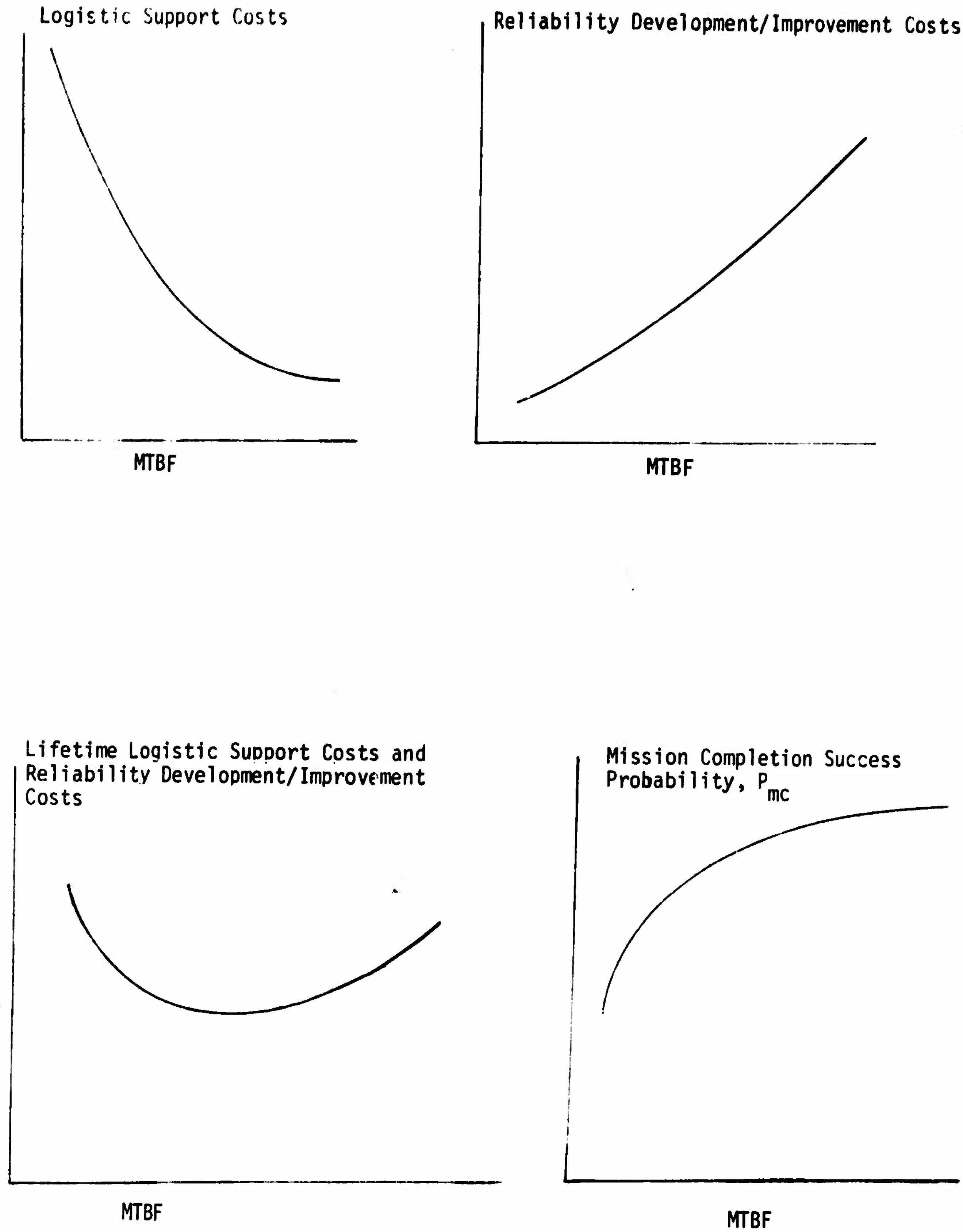


Figure 10. Graphical representation of the Designing to System Performance/ Cost Methodology.

SECTION IV

DESIGNING TO SYSTEM PERFORMANCE/COST MODEL

1. INTRODUCTION

The previous sections have laid the groundwork for development of the Designing to System Performance/Cost (DSPC) model. In this section the detailed DSPC methodology is presented along with numerical examples. A digital computer program listing for the model is presented in Appendix B.

2. STATEMENT OF THE PROBLEM

Consider a system consisting of a certain number of mission critical subsystems. For some subsystems, there are options, each characterized by an acquisition cost, reliability (MTBF), and average cost per repair. The objective is to select one option for each subsystem such that a maximum value of MCSP is achieved at a cost not exceeding some prescribed limit. (Conversely, the problem can be formulated in terms of achieving a prescribed value of MCSP for the least cost.) Cost can be either acquisition cost only or the sum of acquisition costs plus logistic support costs for y years, e.g., y = 10 or 15 year logistic support cost. The methodology can also be applied to existing systems when it is desired to optimize reliability improvement programs.

The optimization procedure developed in this chapter is very simple and easily implemented. The procedure will yield a curve (such as that depicted in Figure 11) consisting of straight line segments connecting vertex points. The curve has the following properties:

a. Each vertex point represents the maximum MCSP achievable at the associated cost.

b. No combination of subsystem options will yield a point above the curve.

The vertex points represent optimal decision alternatives; for each such point the combination of subsystem options is identified which yields that optimal combination of MCSP and cost. It will be shown that going from one vertex point to the next involves a change in only one subsystem option. Therefore, in a sense, intermediate points on the straight line connecting two consecutive vertex points can be realized (on a fleet basis) by equipping only a certain fraction of the fleet with the new option.

3. REQUIRED INPUTS

The following notation is introduced to describe the inputs required to implement the DSPC model (a complete list is given in Appendix B):

N ≡ total number of systems.

N_s ≡ number of mission critical subsystems.

m ≡ average number of missions per month per system.

y ≡ number of years to be considered in the calculation of logistic support costs.

t_i ≡ operating time of i-th subsystem ($i = 1, 2, \ldots, N_s$) during one mission, i.e., duty cycle of i-th subsystem.

α_i ≡ ratio of total operating time to mission operating time.

$T_i = 12ym\alpha_i t_i$ ≡ total y-year operating time of subsystem i.

P_{aij} ≡ probability that a failure of the i-th subsystem during the j-th mission phase will cause an abort of the mission.

$n(i)$ ≡ number of options for the i-th subsystem.

C_{ij} ≡ Cost of the j-th option for the i-th subsystem ($j = 1, 2, \ldots, n(i)$; $i = 1, 2, \ldots, N_s$).

τ_{ij} ≡ lower MTBF for the j-th option for the i-th subsystem.

$\bar{\tau}_{ij}$ ≡ upper MTBF for the j-th option for the i-th subsystem.

CR_{ij} ≡ average cost per repair associated with the j-th option for the i-th subsystem.

As shown in Section II, the MCSP is a function of the duty cycle t_i, the abort probability P_{aij}, and the reliability (MTBF) of each of the N_s mission critical subsystems. The performance/cost tradeoffs arise from the different options available for the subsystems, i.e., for a unit acquisition cost of C_{ij} dollars for the j-th option of subsystem i, the subsystem will have an MTBF of at least τ_{ij} hours and possibly as high as $\bar{\tau}_{ij}$, and the average cost per repair will be CR_{ij}. If one option is selected for each subsystem the MCSP is determined, and the total y-year cost (excluding operating costs) is the sum of the acquisition costs plus the y-year logistic support costs of the subsystems. The y-year logistic support cost for the j-th option for subsystem i is

$$\frac{NT_i}{\tau_{ij}} CR_{ij} \quad . \tag{IV-1}$$

Therefore, the total y-year cost (excluding operating costs) of the j-th option for subsystem i is

$$\bar{C}_{ij} = N\left\{C_{ij} + \frac{T_i}{\tau_{ij}} CR_{ij}\right\} \quad . \tag{IV-2}$$

The options for each subsystem can always be ordered in terms of increasing MTBF such that $\tau_{ij+1} \geq \tau_{ij}$, i.e., the reliability of the (j + 1)st option is equal to or greater than that of the j-th option. This relation is assumed to hold for each subsystem. It also should be mentioned that to optimize with respect to acquisition cost only, the value of y should be set equal to zero.

4. DESCRIPTION OF OPTIMIZATION PROCEDURE

For clarity it is desirable to change notation slightly from that in Section II and to express the MCSP function in slightly different form. Let

τ_i denote the MTBF of the i-th subsystem, e.g., for $i = 1, 2, \ldots, N_s$, τ_i is one of the values τ_{ij}, $j = 0, 1, \ldots, n(i)$. Once the MTBF of each subsystem is specified then the MCSP denoted by P_{mc} is given by

$$P_{mc} = \prod_{i=1}^{N_s} P_i(\tau_i) \quad , \tag{IV-3}$$

where $P_i(\tau_i)$ denotes the probability that the i-th subsystem does not have an abort causing failure. Observe that if the value of τ_i is changed to τ_i' then the resulting MCSP becomes

$$P_{mc}' = P_{mc}\left(\frac{P_i(\tau_i')}{P_i(\tau_i)}\right) . \tag{IV-4}$$

Letting

$$\lambda_i(\tau_i, \tau_i') = \left(\frac{P_i(\tau_i')}{P_i(\tau_i)}\right) , \tag{IV-5}$$

the incremental change in P_{mc} resulting from the MTBF change from τ_i to τ_i' can be written

$$\Delta P_{mc} = P_{mc}' - P_{mc} = P_{mc}\left\{\lambda_i(\tau_i, \tau_i') - 1\right\} \quad . \tag{IV-6}$$

Thus, P_{mc} needs to be calculated only for the baseline system ($\tau_i = \tau_{io}$ for $i = 1, 2, \ldots, N_s$), and any changes in P_{mc} resulting from the selection of a new option can be calculated easily using the above procedure.

It is clear that the optimization problem can be formulated as a zero-one integer linear programming problem, i.e., letting $x_{ij} = 1$ if the j-th option for subsystem i is selected and 0 otherwise, the problem is (for some prescribed cost constraint C) to:

$$\text{maximize } \log P_{mc} = \sum_{i=1}^{N_s} \sum_{j=0}^{n(i)} x_{ij} \log P_i(\tau_{ij}) \quad , \qquad \text{(IV-7a)}$$

subject to

$$\sum_{j=0}^{n(i)} x_{ij} = 1, \; i = 1, 2, \ldots, N_s \qquad \text{(IV-7b)}$$

$$\sum_{i=1}^{N_s} \sum_{j=0}^{n(i)} x_{ij}\overline{C}_{ij} \leq C \quad . \qquad \text{(IV-7c)}$$

Although algorithms exist for solving such zero-one integer problems, they require rather complex computer programs. A much simpler and straightforward optimization procedure will be developed which will yield an optimal curve such as that shown in Figure 11.

To determine the starting point, it is first necessary to calculate the baseline MCSP and cost:

$$P_{mco} = \prod_{i=1}^{N_s} P_i(\tau_{io}) \quad . \qquad \text{(IV-8)}$$

$$C_o = \sum_{i=1}^{N_s} \overline{C}_{io} = N \sum_{i=1}^{N_s} \left\{ C_{io} + \frac{T_i}{\tau_{io}} CR_{io} \right\} \quad . \qquad \text{(IV-9)}$$

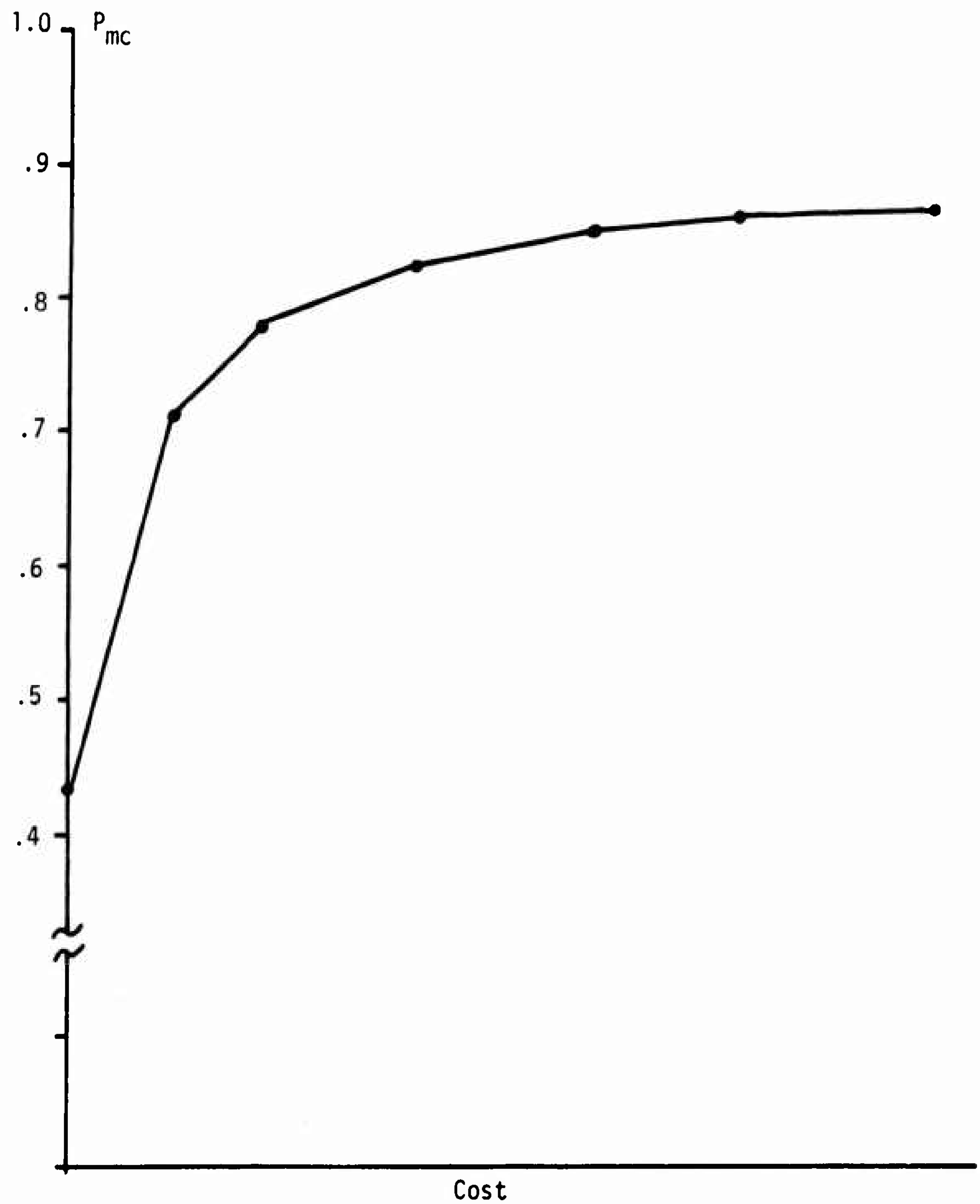


Figure 11. Optimal DSPC Curve.

The next step is to check each subsystem for the possibility of reliability optimization as described in Section III. This means that each subsystem must be checked to determine if there is an option $j > 0$ such that $\overline{C}_{ij} \leq \overline{C}_{io}$. If an option with this property exists, then an MCSP greater than P_{mco} can be achieved at a cost less than the baseline cost since the MTBF's for the options are ordered, i.e., $\tau_{ij+1} \geq \tau_{ij}$. Thus, if for some subsystem i there exists an option $j > 0$ such that $\overline{C}_{ij} \leq \overline{C}_{io}$, then an adjusted baseline (with a higher MCSP at lower cost) is determined as follows:

a. For each subsystem i calculate

$$\min_{0 \leq j \leq n(i)} \left\{ \overline{C}_{ij} \right\} \quad . \tag{IV-10}$$

b. Let m(i) denote the maximum (in case there are several minima) value of j for which $\overline{C}_{ij}$ is a minimum. Let option m(i) be the adjusted baseline for subsystem i.

c. Reject the options $j = 0, 1, \ldots, m(i) - 1$ since these all result in a lower MCSP at higher cost.

The adjusted baseline for the i-th subsystem has an MTBF of $\tau_{im(i)}$ at a cost $\overline{C}_{im(i)}$. The only remaining options for the i-th subsystem are

$$\left\{ \tau_{ij}, \overline{C}_{ij} \right\} , \; m(i) < j < n(i) \quad . \tag{IV-11}$$

For the adjusted baseline system the new values of MCSP and cost are calculated in a manner analogous to that described in equations (IV-8) and (IV-9), or by repeated application of equation (IV-4).

With the adjusted baseline system as the new starting point and each subsystem having the options defined by (IV-11), the optimization procedure can now be developed.

A procedure to be discussed first is the method of steepest slope which can be described as follows:

> At any vertex point the next vertex point is determined by selecting, among the remaining options, that option which maximizes the improvement in MCSP per dollar, i.e., maximizes

$$\frac{\Delta P_{mc}}{\Delta \text{Cost}} \quad . \tag{IV-12}$$

Letting k(i) denote the option for the i-th subsystem at some vertex point then the next vertex point is determined by finding the subsystem i and option j from the maximization

$$\max_{1 \le i \le N_s} \quad \max_{k(i) < j \le n(i)} \left\{ \frac{\lambda_i(\tau_{ik(i)}, \tau_{ij}) - 1}{\overline{C}_{ij} - \overline{C}_{ik(i)}} \right\} \quad . \tag{IV-13}$$

Notice that the value of P_{mc} at the vertex point does not appear in (IV-13) since it enters only as a constant factor as shown by equation (IV-6). Although the method of steepest slope works in many cases, it fails to select the optimal vertex point when there exists a combination of subsystems and options whose total incremental cost is less than the incremental cost of the selected system and whose combined ΔP_{mc} exceeds that of the selected systems. For example, suppose the selected system has Δ cost = 10 and

$$\Delta P_{mc} = P_{mc}(1.2 - 1) = 0.2 P_{mc} \quad .$$

It follows that

$$\frac{\Delta P_{mc}}{\Delta \text{Cost}} = .02 P_{mc} \quad . \tag{IV-14}$$

Suppose there are options for 10 other subsystems each with an incremental cost of 1.0 and with $\Delta P_{mc} = P_{mc}\ (1.0199 - 1)$. Thus, for each of the 10 subsystems

$$\frac{\Delta P_{mc}}{\Delta \mathrm{Cost}} = (.0199)\ P_{mc} \quad , \tag{IV-15}$$

which is less than the slope given by (IV-14). However, if all 10 subsystems are selected their combined effect, by repeated application of equation (IV-6), is

$$\Delta P_{mc} = P_{mc}\ (1.0199^{10} - 1) = .218 P_{mc} \tag{IV-16}$$

with a total incremental cost of 10. This shows that the method of steepest slope does not always select the best option since there can exist a combination of subsystems with smaller costs yielding a better result. However, if all incremental costs were equal the method would work. This suggests a modification of the method of steepest slope which will be described in the next paragraph.

Let k(i) denote the option for the i-th subsystem at some vertex point. For $i = 1, 2, \ldots, N_s$ and $j = k(i) + 1, \ldots, n(i)$, calculate:

$$\text{(a)} \quad \lambda_i(\tau_{ik(i)}, \tau_{ij}) \quad . \tag{IV-17a}$$

$$\text{(b)} \quad \Delta C_{ij} = 1/N \left\{ \overline{C}_{ij} - \overline{C}_{ik(i)} \right\} \quad . \tag{IV-17b}$$

$$\text{(c)} \quad \lambda_{ij} = \left\{ \lambda_i(\tau_{ik(i)}\ ,\ \tau_{ij}) \right\}^{1/\Delta C_{ij}} \quad . \tag{IV-17c}$$

The next subsystem (to be replaced) and its option is determined by selecting i and j such that λ_{ij} is a maximum. The above calculations then have to be repeated only for that subsystem and option which was added (all the other λ_{ij} values remain the same), and the process is continued by selecting the maximum λ_{ij} among the new set.

The procedure described above is equivalent to considering a number ΔC_{ij} of separate subsystems each costing one unit of cost and yielding a relative change in MCSP of

$$\lambda_{ij} - 1 \quad .$$

These ΔC_{ij} pseudo-subsystems, each of 1 unit of cost, have the property that when all ΔC_{ij} are selected then the incremental change in MCSP is

$$\begin{aligned} \Delta P_{mc} &= P_{mc} \left\{ (\lambda_{ij})^{\Delta C_{ij}} - 1 \right\} \\ &= P_{mc} \left\{ \lambda_i(\tau_{ik(i)} , \tau_{ij}) - 1 \right\} \quad , \end{aligned} \tag{IV-18}$$

and the incremental cost is ΔC_{ij}. In other words, the selection of all ΔC_{ij} of these pseudo-subsystems is equivalent to selecting the j-th option of subsystem i. It remains to be shown that this selection process is optimal in the sense described above in the statement of the problem.

If the value of MCSP at a vertex point is P_{mc} and if the i-th subsystem with option j is chosen for the next vertex point, then the value of MCSP at that vertex point is

$$(\lambda_{ij})^{\Delta C_{ij}} P_{mc} \quad . \tag{IV-19}$$

with an incremental cost ΔC_{ij}. For an incremental cost $\Delta C \leq \Delta C_{ij}$ the pseudo-path between the two consecutive vertex points has the value

$$(\lambda_{ij})^{\Delta C} P_{mc} = \lambda_i(\tau_{ik(i)} , \tau_{ij}) P_{mc} \quad . \tag{IV-20}$$

It is easily shown that the value (IV-20) lies below the straight line connecting the two vertex points. Furthermore, the pseudo-path between two consecutive vertex points has monotonically increasing slope and has the form depicted by the dashed curve in Figure 12.

Suppose the selection process leads to n ordered values of the λ_{ij}. Let these n values of λ_{ij} and the corresponding incremental costs be denoted by

$$\lambda_1 > \lambda_2 > \ldots > \lambda_n \quad . \tag{IV-21a}$$

$$\Delta C_1, \Delta C_2, \ldots, \Delta C_n \quad . \tag{IV-21b}$$

For the purpose of proving that the procedure is optimal, the assumption of strict inequality in (IV-21a) is justified. Assume that for some cost C there exists a combination of subsystem options with a total cost C and with an MCSP above the curve generated by the procedure (IV-17). In terms of the ΔC_i defined in (IV-21) the cost C can be written

$$C = \Delta C_1 + \Delta C_2 + \ldots + \Delta C_k + r \quad , \tag{IV-22}$$

where

$$0 < r \leq \Delta C_{k+1} \text{ and } k \leq n \quad .$$

In other words, C lies between the costs corresponding to the k-th and (k + 1)-th vertex. In Figure 13 the dashed pseudo-path leading from the k-th to the (k + 1)-th vertex is shown. In reaching the point (on the pseudo-path) corresponding to cost C, the greatest C values of λ were selected to yield the MCSP of

$$P_{Co}(\lambda_1)^{\Delta C_1}(\lambda_2)^{\Delta C_2}\ldots(\lambda_k)^{\Delta C_k}(\lambda_{k+1})^{r} \quad , \tag{IV-23}$$

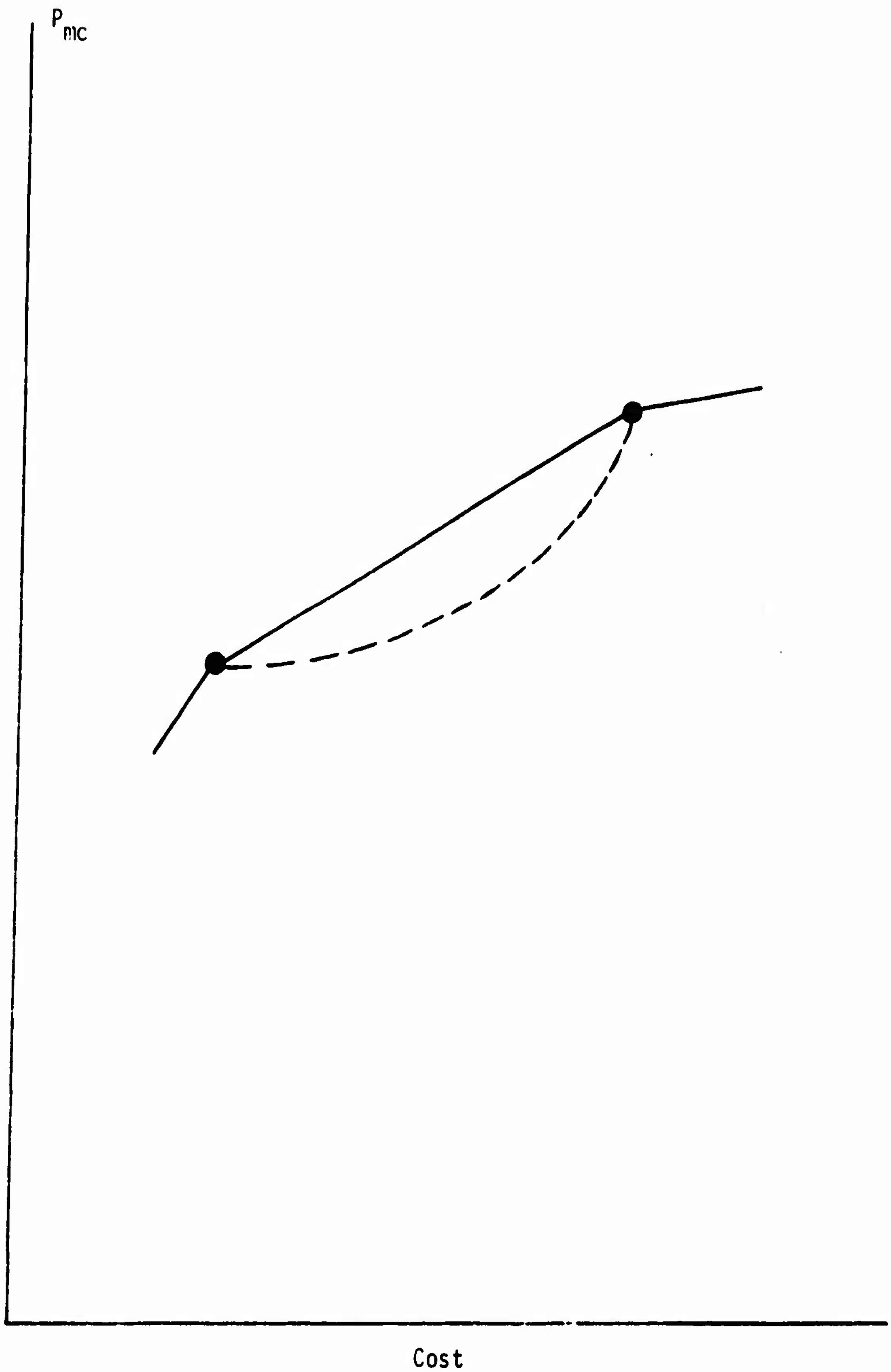


Figure 12. Pseudo-Path between Two Vertex Points.

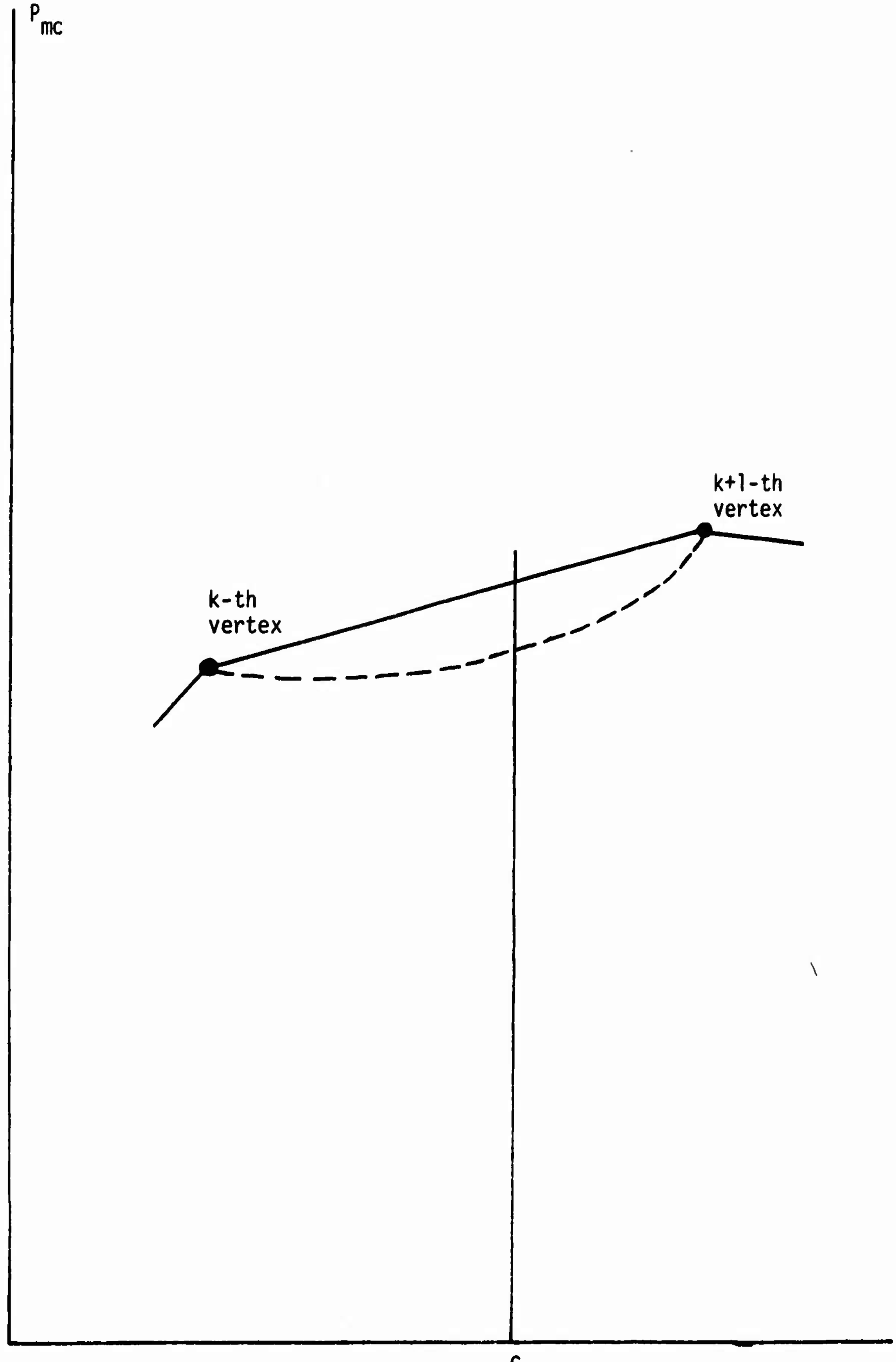


Figure 13. Optimal P_{mc} at Cost C.

where P_{Co} denotes the MCSP of the adjusted baseline system. Any other combination of subsystem options with total cost C requires that some of its corresponding λ values be different from those in (IV-23). Consequently, for any other combination some of the λ values in (IV-23) would be replaced by smaller values since (IV-23) contains the C greatest λ values. Thus, any other combination of subsystem options with cost C leads to an MCSP lying not only below the straight line segment but also below the dashed pseudo-path. This completes the proof.

This optimization procedure has been developed using the higher confidence reliabilities τ_{ij} rather than the upper limit $\bar{\tau}_{ij}$. Once the optimum curve has been obtained, its upper limit can be obtained by substituting $\bar{\tau}_{ij}$ for τ_{ij} in the appropriate equations. However, this curve is not necessarily optimal since the ordering and selection process for the options would in general be different when the optimization procedure is with respect to the $\bar{\tau}_{ij}$. If a high risk program is to be considered, the model should be exercised in both ways, i.e., determine the upper limit of the optimum curve based on high confidence MTBFs and optimize with respect to the upper limit MTBFs.

5. NUMERICAL EXAMPLE

To illustrate the procedure described in the previous section it is instructive to carry through the calculations in detail for a specific example.

Consider one system (N = 1) consisting of 3 subsystems (N_s = 3), each subsystem having 3 options (n(1) = n(2) = n(3) = 3). The cost to be considered will be the sum of acquisition costs plus 15-year logistic support costs (y = 15). An average of 10 missions per month (m = 10) is assumed. The duty cycle t_i, abort probability P_{ai}, and 15-year operating time T_i for each subsystem is presented in Table VI.

Table VI

SUBSYSTEM OPERATING CHARACTERISTICS

SUBSYSTEM (i)	t_i (hrs)	P_{ai}	$T_i = 1800t_i$ (hrs)
1	3	0.2	5400
2	0.5	1.0	900
3	2	0.8	3600

The subsystem options are defined in Table VII.

Table VII

SUBSYSTEM OPTIONS

	OPTION 0 (Baseline)				OPTION 1				OPTION 2			
i	C_{i0}	τ_{i0}	$\overline{\tau}_{i0}$	CR_{i0}	C_{i1}	τ_{i1}	$\overline{\tau}_{i1}$	CR_{i1}	C_{i2}	τ_{i2}	$\overline{\tau}_{i2}$	CR_{i2}
1	3	10	15	.04	6	16	20	.04	15	30	36	.05
2	1	5	8	.10	2	10	15	.20	14	25	30	.20
3	2	8	10	.07	8	12	16	.09	20	22	30	.15

The unit of cost assumed in this example is \$10,000. Using the costs and MTBF values for each option given by Table VII and using the operating characteristics given in Table VI, the values $\overline{C}_{ij}$ and $P_i(\tau_{ij})$ are calculated using equations (IV-2) and

$$P_i(\tau_{ij}) = \exp\left(\frac{-t_i P_{ai}}{\tau_{ij}}\right) \quad . \qquad \text{(IV-24)}$$

These results are presented in Table VIII.

Table VIII

COST AND MISSION PERFORMANCE FOR EACH SUBSYSTEM OPTION

	OPTION 0		OPTION 1		OPTION 2	
i	$P_i(\tau_{i0})$	$\overline{C}_{i0}$	$P_i(\tau_{i1})$	$\overline{C}_{i1}$	$P_i(\tau_{i2})$	$\overline{C}_{i2}$
1	.9418	24.6	.9632	19.5	.9802	24.0
2	.9048	19.0	.9512	20.0	.9802	21.2
3	.8187	33.5	.8752	35.0	.9299	44.6

For the baseline system:

$$P_{mco} = \prod_{i=1}^{3} P_i(\tau_{i0}) = .6976 \quad . \qquad \text{(IV-25)}$$

$$C_o = \sum_{i=1}^{3} \overline{C}_{i0} = 77.1 \quad . \qquad \text{(IV-26)}$$

Checking each subsystem for reliability optimization shows that Option 1 for Subsystem 1 should replace Option 0 since it yields a higher MCSP at lower cost. In other words, the increase in acquisition cost in going from Option 0 to Option 1 is more than compensated for by the savings in logistic support cost. Thus, the adjusted baseline system consists of Option 1 for Subsystem 1 and Option 0 for Subsystems 2 and 3. This combination of options will be denoted by (1, 0, 0). The MCSP and cost of the adjusted baseline system is

$$P_{mc} = .6976 \frac{.9632}{.9418} = .7135 \quad .$$

$$\text{Cost} = 72 \quad . \tag{IV-27}$$

With the adjusted baseline system established, the optimization procedure described by equations (IV-17) can now be applied. Since Option 1 has been selected for Subsystem 1, the value of k(1) is set equal to 1. The values of k(2) and k(3) are 0. Table IX lists the values of

$$\lambda_i(k(i)) = \max_{j > k(i)} \left\{ \lambda_{ij} \right\} \quad , \tag{IV-28}$$

from which the optimal options can be determined.

Table IX

EVALUATION OF OPTIONS

SUBSYSTEM i	k(i) = 0		k(i) = 1	
	$\lambda_i(0)$	NEXT ELIGIBLE OPTION	$\lambda_i(1)$	NEXT ELIGIBLE OPTION
1	--	--	1.0039	2
2	1.0513	1	1.0253	2
3	1.0455	1	1.0063	2

Starting with the combination of options (1, 0, 0) the next vertex point is determined from Table IX by finding the maximum of $\lambda_i(k(i))$ where $k(1) = 1$, $k(2) = k(3) = 0$. This maximum is 1.0513 which means Option 1 for

Subsystem 2 should be added to yield (1, 1, 0). The next option is determined from the maximum $\lambda_i(k(i))$ for $k(1) = k(2) = 1$, $k(3) = 0$. This gives (1, 1, 1). Proceeding in this manner yields the sequence

$$(0, 0, 0) \rightarrow (1, 0, 0) \rightarrow (1, 1, 0) \rightarrow (1, 1, 1) \rightarrow$$
$$(1, 2, 1) \rightarrow (1, 2, 2) \rightarrow (2, 2, 2) \quad . \qquad \text{(IV-29)}$$

Using Table VIII to determine the corresponding P_{mc} and cost of each configuration yields the results shown in Table X.

Table X

OPTIMAL MCSP AND COSTS

CONFIGURATION	P_{mc} (lower)	COST	P_{mc} (upper)	$\overline{\text{COST}}$
(0, 0, 0)	.6976	77.1	.7691	56.8
(1, 0, 0)	.7135	72.0	.7768	56.2
(1, 1, 0)	.7501	73.0	.7998	58.0
(1, 1, 1)	.8019	74.5	.8492	59.0
(1, 2, 1)	.8263	75.7	.8635	65.0
(1, 2, 2)	.8779	85.3	.9049	74.8
(2, 2, 2)	.8934	89.8	.9171	80.5

Figure 14 shows the optimal MCSP vs cost curve. The curve corresponding to the upper values of MCSP is not plotted. For this simple example, there are $3^3 = 27$ possible combinations of options, and for purposes of illustration all combinations were calculated and are plotted in Figure 14.

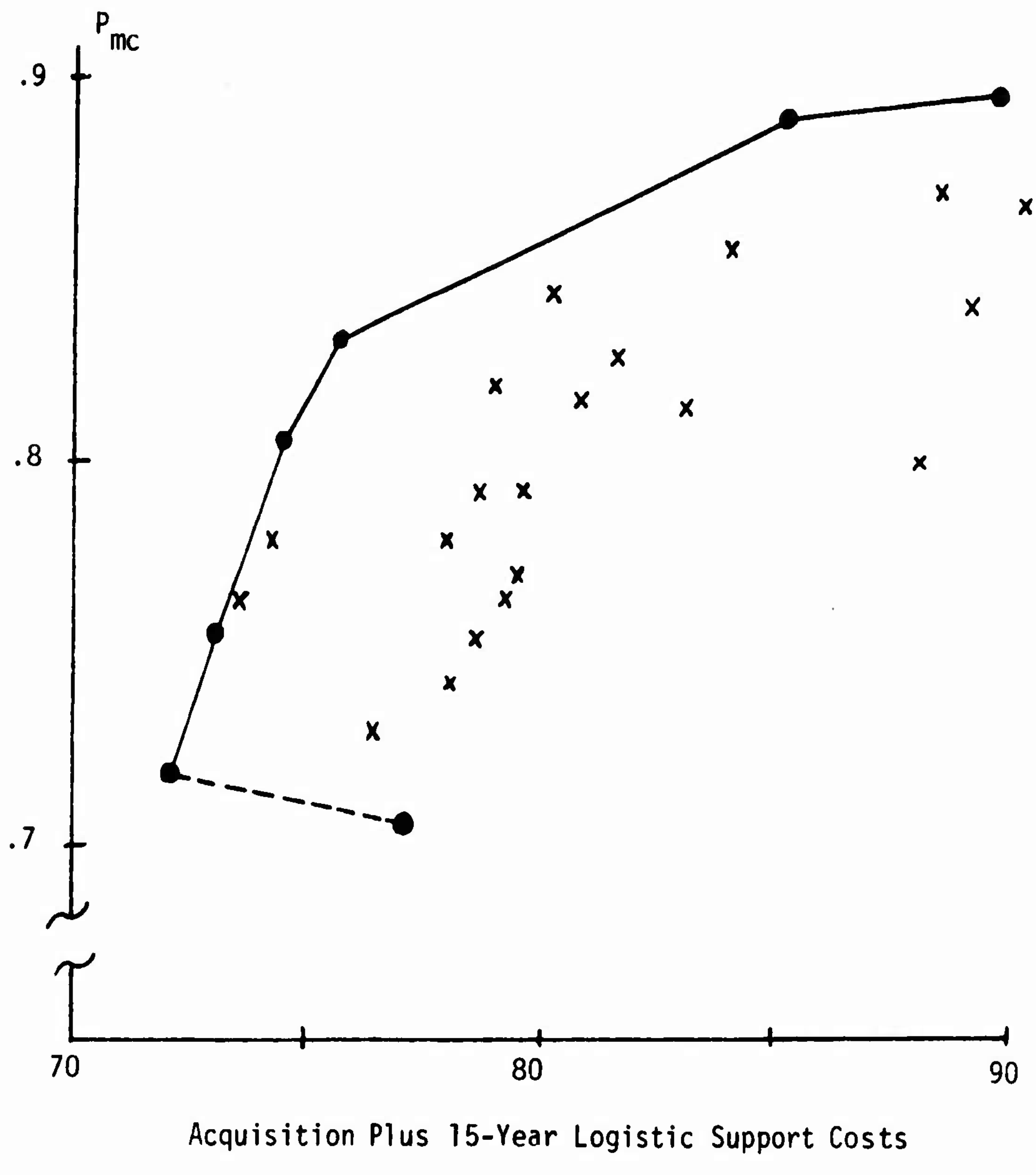


Figure 14. DSPC Example.

To demonstrate the necessity of an efficient algorithm for calculating the optimal MCSP vs cost curve, it is instructive to discuss the case of the A-7D. For this aircraft a total of 36 mission critical subsystems were identified. If, for example, during the planning phase there were 3 options for each subsystem then the total number of combinations of subsystem options would be

$$3^{36} = 1.5 \times 10^{17} \quad . \tag{IV-30}$$

Even if a computer required only 1 millisecond to calculate the MCSP and cost associated with each combination, a total computer time of 4.8 million years would be required to compute all combinations.

It is easily shown that the procedure described in this chapter requires at most the calculation of

$$\sum_{i=1}^{N_s} \frac{n(i)\,\{n(i) - 1\}}{2} \tag{IV-31}$$

values of the λ_{ij}. The ordering of these λ_{ij} values then gives the optimal options. For the above mentioned A-7D example of 36 subsystems each having 3 options, the maximum number of calculations of the λ_{ij} values is

$$36\,\frac{3(2)}{2} = 108 \quad . \tag{IV-32}$$

It is instructive to apply the optimization procedure to the above example when the system is optimized with respect to acquisition costs only. The results of the optimization procedure lead to the following sequence of configurations:

$$(0, 0, 0) \rightarrow (0, 1, 0) \rightarrow (0, 1, 1) \rightarrow (1, 1, 1) \rightarrow$$
$$(1, 1, 2) \rightarrow (1, 2, 2) \rightarrow (2, 2, 2) \quad . \qquad \text{(IV-33)}$$

The sequence of system configurations (IV-33), optimized for acquisition costs, differs significantly from the sequence (IV-29) which is optimal for the sum of acquisition and 15-year logistic support costs. Table XI shows the acquisition cost and MCSP for the sequence of configurations (IV-33).

Table XI

OPTIMAL MCSP AND ACQUISITION COSTS

CONFIGURATION	P_{mc} (lower)	ACQUISITION COST
(0, 0, 0)	.6976	6
(0, 1, 0)	.7334	7
(0, 1, 1)	.7840	13
(1, 1, 1)	.8019	16
(1, 1, 2)	.8520	28
(1, 2, 2)	.8779	40
(2, 2, 2)	.8934	49

It is instructive to investigate the consequences (in terms of total 15-year costs) of designing a system to acquisition cost. For this purpose the total 15-year costs were calculated for the configurations identified in Table XI. In Figure 15 the results (encircled points) are compared with the curve which resulted from optimizing to total 15-year cost. As shown in

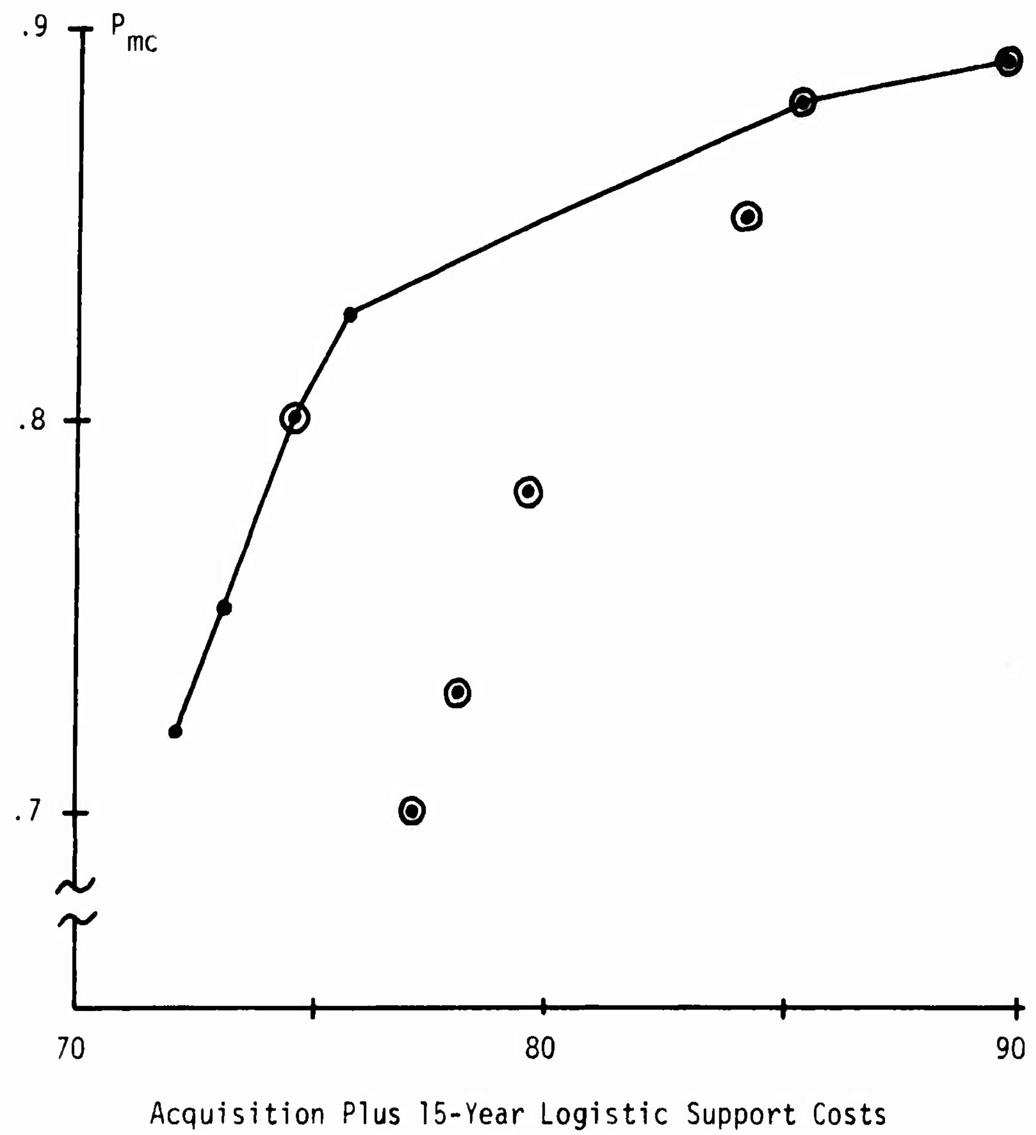


Figure 15. Comparison of Acquisition Cost Optimization with Total Cost Optimization.

the figure, most of the encircled points lie far off the optimal curve. This large discrepancy between the results for the two cases indicates the implications in designing to acquisition cost rather than total 15-year cost.

To demonstrate how the optimization procedure treats redundancy options it is now assumed that Subsystem 3 can be redundant. The operating characteristics and subsystem options defined in Tables VI and VII remain the same but Subsystem 3 can also have redundancy with Option 0, Option 1, or Option 2. Standby redundancy with two identical units (primary and backup) is assumed. The optimization procedure will identify when (at what cost level) redundancy should be considered and also identify which option should be redundant.

The values of the cost and mission performance parameters listed in Table VIII remain the same; however, the corresponding values must be calculated for redundancy of Option 0, Option 1, and Option 2 with Subsystem 3. Using equations (II-14), (II-20), and (II-26) these values are calculated and listed in Table XII.

Table XII

COST AND MISSION PERFORMANCE FOR STANDBY REDUNDANCY OPTIONS FOR SUBSYSTEM 3

OPTION 0 REDUNDANT		OPTION 1 REDUNDANT		OPTION 2 REDUNDANT	
$P_{3R}(\tau_{i0})$	$\overline{C}_{3R0}$	$P_{3R}(\tau_{i1})$	$\overline{C}_{3R1}$	$P_{3R}(\tau_{i2})$	$\overline{C}_{3R2}$
.9825	38.7	.9919	44.8	.9975	65.4

Starting with the baseline for Subsystem 3 (i.e., $k(3) = 0$) Table IX shows that the next eligible nonredundant option for Subsystem 3 is Option 1 with

$\lambda_3(0) = 1.0455$. This value must be compared with corresponding λ values for the redundancy options. Going from Option 0 to Option OR (with redundancy) yields a λ value of

$$1.0357. \tag{IV-34}$$

Going from Option 0 to Options 1 or 2 with redundancy yields the λ values

$$1.0171 \quad \text{and} \quad 1.0062 \quad . \tag{IV-35}$$

Since all λ values for the reliability options are less than $\lambda_3(0) = 1.0455$ the next eligible option is Option 1 without redundancy. After Option 1 is selected Table IX shows that the next eligible nonredundant option is Option 2 with $\lambda_3(1) = 1.0063$. This value must be compared with the λ values associated with going from Option 1 to redundant Option 0, 1, and 2. These values are 1.0322, 1.0128, and 1.0043. Thus, the next eligible Option is Option 1 with redundancy. For the redundancy options of Subsystem 3 the values corresponding to those of Table IX are given in Table XIII.

Table XIII

EVALUATION OF OPTIONS FOR STANDBY REDUNDANCY
OPTIONS FOR SUBSYSTEM 3

SUBSYSTEM i	k(i) = 0		k(i) = 1		k(i) = 2	
	$\lambda_i(0)$	NEXT ELIGIBLE OPTION	$\lambda_i(1)$	NEXT ELIGIBLE OPTION	$\lambda_i(2)$	NEXT ELIGIBLE OPTION
1	--	-	1.0039	2	--	-
2	1.0513	1	1.0253	2	--	-
3	1.0455	1	1.0322	0 + Redundancy	1.0015	1 + Redundancy

The sequence for selecting options is then

$$(0, 0, 0) \rightarrow (1, 0, 0) \rightarrow (1, 1, 0) \rightarrow (1, 1, 1) \rightarrow (1, 1, 0R) \rightarrow$$
$$(1, 2, 0R) \rightarrow (2, 2, 0R) \rightarrow (2, 2, 1R) \rightarrow (2, 2, 2R) \quad . \qquad \text{(IV-36)}$$

Observe that redundancy was not selected until late in the sequence. The optimal MCSP and costs are presented in Table XIV.

Table XIV

OPTIMAL MCSP AND COSTS FOR STANDBY REDUNDANCY OPTIONS FOR SUBSYSTEM 3

CONFIGURATION	P_{mc}	COST
(0, 0, 0)	.6976	77.1
(1, 0, 0)	.7135	72.0
(1, 1, 0)	.7501	73.0
(1, 1, 1)	.8019	74.5
(1, 1, 0R)	.9002	78.2
(1, 2, 0R)	.9277	79.2
(2, 2, 0R)	.9440	83.7
(2, 2, 1R)	.9530	89.8
(2, 2, 2R)	.9584	110.4

These results illustrate the fact that even if a subsystem can be redundant it does not follow that redundancy is the optimal decision.

6. SUMMARY

The DSPC methodology represents a new and innovative approach to system acquisition, and preliminary results indicate that this technique will provide very valuable information to the decision-maker. The DSPC model is compatible with designing to system cost, or performance, or both. Once total system reliability specifications are established, each individual subsystem has a corresponding installed reliability and cost goal, which allows realistic and continuous evaluation and adjustments as the subsystem is developed to maturity.

It should be pointed out that although the model has been formulated in terms of optimizing the performance of the total system, the methodology can also be profitably applied to individual subsystems. For this case, the subsystem is considered as the total system and its components are considered as the subsystems. Then the reliability optimization procedures are applied such that component reliability levels are established such that the desired subsystem reliability is achieved.

As indicated above, the DSPC methodology appears to have great potential in the system acquisition process. However, there are two important caveats. First, if the required data are not available, it will be impossible to design to required levels of performance at minimum cost. Second, assuming the necessary data are available, if DSPC techniques cannot be incorporated into system acquisition contracts, then it will be impossible to achieve required levels of performance at minimum cost except on a chance basis.

Preliminary investigations by OAS indicate that a great deal of data are available (especially at AFLC Air Materiel Areas). In some cases, rough estimates are necessary, but these can be refined as more emphasis is placed establishing and maintaining a DSPC data bank. The means of implementing DSPC techniques in contractual requirements are well beyond the scope of OAS efforts in life cycle cost analysis, but these means must be found if the full potential of the methodology is to be realized.

SECTION V

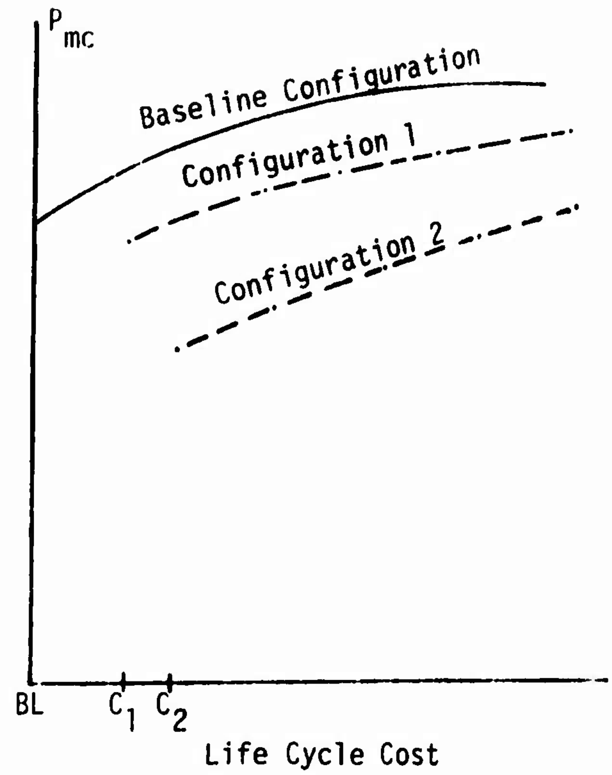
DESIGNING TO SYSTEM PERFORMANCE/COST/EFFECTIVENESS

1. INTRODUCTION

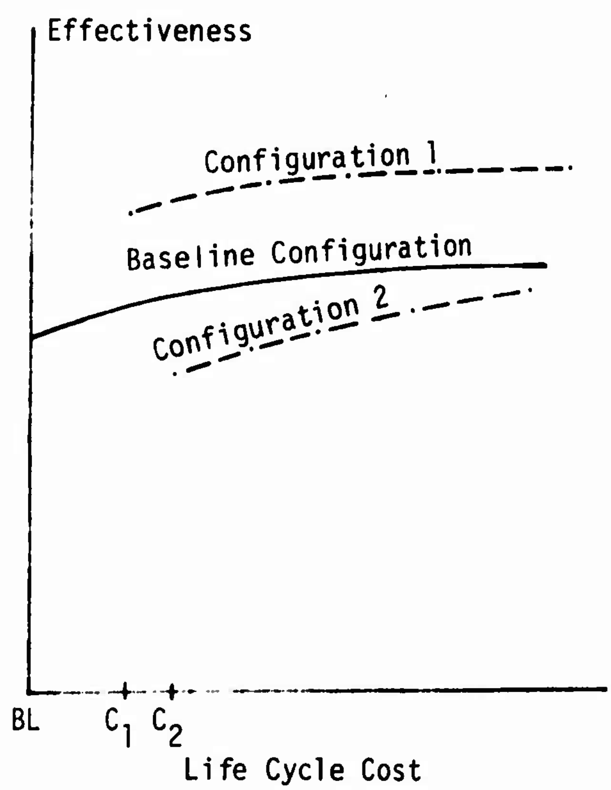
In developing the DSPC model the objective function was mission completion success probability, and mission effectiveness was not considered. It was tacitly assumed that mission effectiveness met mission requirements. As shown in the previous section, the DSPC approach can be a very valuable management tool; but for a more complete system evaluation, mission effectiveness must also be considered. By combining the results of a DSPC model with those from the appropriate mission effectiveness model, information can be generated allowing the decision-maker to more effectively evaluate the system. This is particularly important when system configuration changes are being considered, or when there are competing subsystems during system development. This section presents a hypothetical example illustrating performance/cost/effectiveness interactions.

2. PERFORMANCE/COST/EFFECTIVENESS INTERACTIONS

Figure 16 provides an overall viewpoint of the methodology developed in this study. The graphs show probability of mission completion and mission effectivensss as a function of life cycle costs for three configurations - a baseline configuration and two other configurations in which subsystems have been added in order to increase mission effectiveness. (Effectiveness is defined as some measure as to how well a system accomplishes its mission. For example, for weapon systems it is usually some function of weapons delivery accuracy or targets killed, while for transport aircraft it would generally depend on amount of cargo delivered.)



a. Performance



b. Effectiveness

Figure 16. Performance/Cost/Effectiveness Interactions.

For a given system configuration a major way of improving effectiveness is by improving performance, i.e., subsystem reliabilities. Assuming that optimum subsystem reliability levels have already been established (reliability improvement costs = logistic support cost savings), additional reliability improvement can only be achieved with additional reliability improvement cost and hence, increased life cycle costs. In Figure 16, the increase in P_{mc} due to the reliability improvement is translated into increased mission effectiveness since P_{mc} is one of the principal parameters in determining mission effectiveness. (Reliability improvement also increases system availability which is another principle parameter in determining mission effectiveness.) Mission effectiveness can also be improved by adding on other subsystems, for example, adding subsystems which improve weapon delivery accuracy. As shown in Figure 16-a, additional subsystems increase life cycle costs (additional acquisition costs plus increased logistic support costs) and decrease system performance (overall system reliability is lowered). However, these detrimental effects may be offset by increases in mission effectiveness. This is shown on the Configuration 1 curves in Figure 16-b. On the other hand, if the potential benefits of a configuration change are negated by a decrease in system performance (P_{mc}), then the modification results in mission effectiveness below the baseline level as shown in the Configuration 2 curves in Figure 16-b.

This type of analysis makes it possible for a decision-maker to readily evaluate his options. For example, in Figure 16-b if available funds are less then C_1, then the baseline configuration is the only option. If additional funds are available, Configuration 1 is the preferred option while Configuration 2 is never in contention.

In the next section examples of two measures of effectiveness for fighter aircraft are presented.

SECTION VI

MEASURES OF EFFECTIVENESS FOR FIGHTER AIRCRAFT

1. INTRODUCTION

Previous sections have developed performance/cost relationships, and the last section presented an overview of analyzing performance/cost/effectiveness interactions. It is not possible to develop generalized relationships between performance/cost and effectiveness since effectiveness depends on particular systems and particular missions. Because of this dependence on particular systems and missions, it is sometimes quite difficult to develop valid measures of effectiveness for given systems and missions. Each system-mission combination must be examined, and the system and mission parameters scrutinized to see if meaningful measures of effectiveness can be developed. In this section two measures of effectiveness for fighter aircraft are developed. These measures appear to have great potential in fighter aircraft evaluations.

2. AIR-TO-GROUND FIGHTERS (TARGETS KILLED)

a. Characteristic Effectiveness Parameters. The utility of a tactical interdiction aircraft is dependent upon the following:

(1) Availability.

(2) Probability of reaching target without a critical subsystem aborting the mission.

(3) Kill potential (e.g., number of targets destroyed per successful sortie).

(4) Probability of survival.

The availability of an aircraft depends upon the frequency of repairs and the average repair time (time to restore). The probability of no abort is dependent upon the number, complexity, and reliability of the mission critical subsystems. The kill potential depends upon the number and type of weapon carried, acquisition probability, delivery accuracy, and target type. Survival probability is dependent upon the strength and type of enemy defenses and such aircraft characteristics as ECM, radar cross section, IR signature, armor, and other protective measures.

The worth of an aircraft cannot be assessed by considering any one of the above factors individually. All of the factors must be considered simultaneously to account for their interaction. In this section measures of effectiveness of an aircraft are developed which quantitatively account for the interaction of the characteristic effectiveness parameters.

For most mission types, an aircraft will be sent on repeated sorties provided it survives; thus, any valid measure of effectiveness must account for the cumulative effect of repeated sorties. It is also clear, and will be shown quantitatively, that survivability is of the utmost importance since it determines the average number of sorties an aircraft can complete.

If a particular scenario is specified, then for a given aircraft the characteristic effectiveness parameters serve to characterize that aircraft and scenario. Since actual scenarios change sortie by sortie, the determination of aircraft performance over repeated sorties requires that the characteristic parameters be specified for each sortie. Such a detailed specification would introduce a high level of arbitrariness leading to an unsuitable measure of a system's worth. However, to obtain a measure (not a predictor) of the effectiveness of an aircraft in a given scenario it seems reasonable to keep the scenario fixed (fixed characteristic parameters) and to determine the cumulative effectiveness if the aircraft flies repeated sorties (when it survives) in that fixed scenario. This is the basic idea underlying the measures of effectiveness developed in this section.

The first measure developed is the expected number of targets destroyed if the aircraft flies up to S sorties in a fixed scenario, i.e., aircraft flies repeated sorties (up to a maximum of S) if it survives. The next measure is the lifetime targets destroyed, i.e., aircraft flies repeated sorties as long as it survives. However, the expected number of targets destroyed during the lifetime of an aircraft may not be the prime measure of effectiveness since there are situations in which it is more important to know the effectiveness of an aircraft over, for instance, a 10 or 20 day period. The final measure of effectiveness developed is the expected number of targets destroyed as a function of time, which yields targets destroyed over any prescribed time period.

Although the discussion is in terms of tactical interdiction aircraft, the kill potential can be redefined (for example, in terms of cargo tonnage delivered or enemy aircraft destroyed) to account for airlift, counterair, or other type aircraft.

b. <u>Lifetime Destruction</u>. The definitions listed below will facilitate the mathematical developments contained in this section.

P_{s1} = Probability aircraft survives to release its weapons on target.

P_{s2} = Probability aircraft survives return trip after weapons are released.

P_c = Probability aircraft reaches target and releases weapons without an abort causing failure given that it survives.

P_{sa} = Probability aircraft aborts before releasing weapons and survives the return trip.

c = "Kill Potential" = expected number of targets destroyed after aircraft reaches the target area.

P_s = Single sortie survival probability.

S = Number of sorties aircraft flies (if it survives).

T(S) = Expected number of targets destroyed after S sorties.

T_K = Expected number of targets destroyed during the "lifetime" of the aircraft, i.e., $S \to \infty$.

For a single sortie, the expected number of targets destroyed by an aircraft is

$$T(1) = \rho P_c P_{s1} \qquad \text{(VI-1)}$$

The main problem in this section is to determine the expected number of targets destroyed if the aircraft flies a maximum of S sorties (if it survives). The time required to complete S sorties is treated in the next section. The probability P_i that the aircraft starts its i-th sortie ($i \leq S$) is equivalent to the probability it survives the first i-1 sorties. Therefore,

$$P_i = \left\{ P_{s1} P_c P_{s2} + P_{sa} \right\}^{i-1} = P_s^{i-1}, \ (i = 1,2,\ldots S), \qquad \text{(VI-2)}$$

where P_s denotes the single sortie survival probability. The expected damage from the i-th sortie is

$$P_i \rho P_c P_{s1} = P_s^{i-1} \rho P_c P_{s1} \ . \qquad \text{(VI-3)}$$

Therefore, it follows that the expected number of targets destroyed after S sorties is

$$T(S) = \sum_{i=1}^{S} P_i \rho P_c P_{s1}$$

$$= \rho P_c P_{s1} \sum_{i=1}^{S} P_s^{i-1} = \rho P_c P_{s1} \left\{ \frac{1 - P_s^S}{1 - P_s} \right\} \ . \qquad \text{(VI-4)}$$

Letting $S \to \infty$ in equation (VI-4) it follows that the expected number of targets destroyed during the lifetime of the aircraft is

$$T_K = \frac{\rho P_c P_{s1}}{1 - P_s} \quad . \tag{VI-5}$$

Of course, if for any reason there is an upper limit to the number of sorties the aircraft would fly, then this number should be used in equation (VI-4) to determine the expected damage during the useful lifetime of the aircraft.

The expression (VI-5) for lifetime destruction was derived under the assumptions that the aircraft flies repeated sorties as long as it survives and that the scenario remains the same for each sortie. It is important to point out that this measure of effectiveness has another interpretation. Suppose $N(N = 1, 2, 3, \ldots)$ aircraft each fly one sortie where the parameters ρ, P_c, P_{s1}, and P_s are the same for each aircraft. The expected number of targets destroyed by the N aircraft is

$$N \rho P_c P_{s1} \quad . \tag{VI-6}$$

The expected number of aircraft lost is

$$N(1 - P_s) \quad . \tag{VI-7}$$

The ratio of the quantities (VI-6) and (VI-7) yields a measure of targets destroyed per aircraft lost (exchange ratio) equal to

$$\frac{\rho P_c P_{s1}}{1 - P_s} \quad , \tag{VI-8}$$

which is independent of the number of aircraft. This exchange ratio is identical to expression (VI-5) for lifetime targets killed.

The expected number of sorties completed during the lifetime of an aircraft is

$$\langle S \rangle = \sum_{j=1}^{\infty} jP_s{}^j(1 - P_s) = \frac{P_s}{1 - P_s} \quad . \qquad \text{(VI-9)}$$

This measure is further discussed in the examples in Section VI-2d.

Although the measures (VI-4) and (VI-5) are useful indicators of the effectiveness of an aircraft, they do not reflect the time rate of damage. This is the subject of the next section.

c. <u>Targets Killed as Function of Time</u>. Equation (VI-4) gives the expected number of targets destroyed after S sorties. However, in evaluating the effectiveness of an aircraft, it is also essential to determine the expected time required for the S sorties. This time depends, of course, upon the mission time T_m and also upon the time required to make repairs.

If the aircraft completes S sorties then the expected number of repairs is

$$\frac{ST_m}{\tau_s} \quad , \qquad \text{(VI-10)}$$

where τ_s is the MTBF of the total aircraft system.

Therefore, the expected total repair time is

$$\frac{ST_m t_r}{\tau_s} \quad , \qquad \text{(VI-11)}$$

where t_r is the mean time to restore. If Δt denotes the average time for normal service actions, e.g., refuel and reload, then the expected time to complete S sorties is

$$t(s) = S\left\{T_m + \frac{T_m t_r}{\tau_s} + \Delta t\right\} \quad . \tag{VI-12}$$

If the service actions can be performed while repairs are being made, then Δt in equation (VI-12) should be replaced by

$$\min\left\{\Delta t - \frac{T_m t_r}{\tau_s} , 0\right\} \quad . \tag{VI-13}$$

The Reliability Engineering Handbook, (Reference 4), defines availability as

$$A = \frac{1}{1 + \frac{t_r}{\tau_s}} \quad . \tag{VI-14}$$

From this it follows that

$$\frac{t_r}{\tau_s} = \frac{1}{A} - 1 \quad . \tag{VI-15}$$

Therefore, equation (VI-12) becomes

$$t(s) = S\left\{\frac{T_m}{A} + \Delta t\right\} \quad . \tag{VI-16}$$

Equations (VI-4) and (VI-16) provide the expected number of targets destroyed as a function of time.

In the following section, examples will be given to show how the individual characteristic parameters associated with an aircraft interact in determining the effectiveness of an aircraft.

d. Examples.

a. Lifetime Sorties.

Figure 17 shows the expected number of sorties completed during the lifetime of an aircraft as a function of survival probability. Since the lifetime targets killed T_K is a constant factor multiplied by life-time sorties, the curve for T_K has the same shape as the curve in Figure 17. Of course, the curve cannot be extended indefinitely since there is an upper limit based upon the service life of the aircraft or other such factors. Several conclusions are apparent:

(1) Conditions resulting in survival probabilities below .95 are probably unacceptable in most cases since lifetime sorties is less than 19.

(2) Small improvements in survival probability in the region $P_s < .98$ result in a small increase in lifetime sorties. However, in the region of high P_s (e.g., $P_s > .98$) any small increase in P_s results in a dramatic increase in lifetime sorties. For example, the small increase in P_s from .99 to .995 more than doubles the number of lifetime sorties (from 99 to 199).

(3) Survival probability can be, by far, the most dominant factor in determining the effectiveness of aircraft.

To appreciate the magnitude of the numbers involved, it is instructive to consider a historical but recent engagement in a severe environment where U.S. aircraft flew 1000 sorties against heavily defended targets. During this period, 26 U.S. aircraft were lost. The survival probability in this case was $P_s = 0.974$ which is on the low part of the curve in Figure 17. Under such conditions the average number of sorties per aircraft is only 37.5.

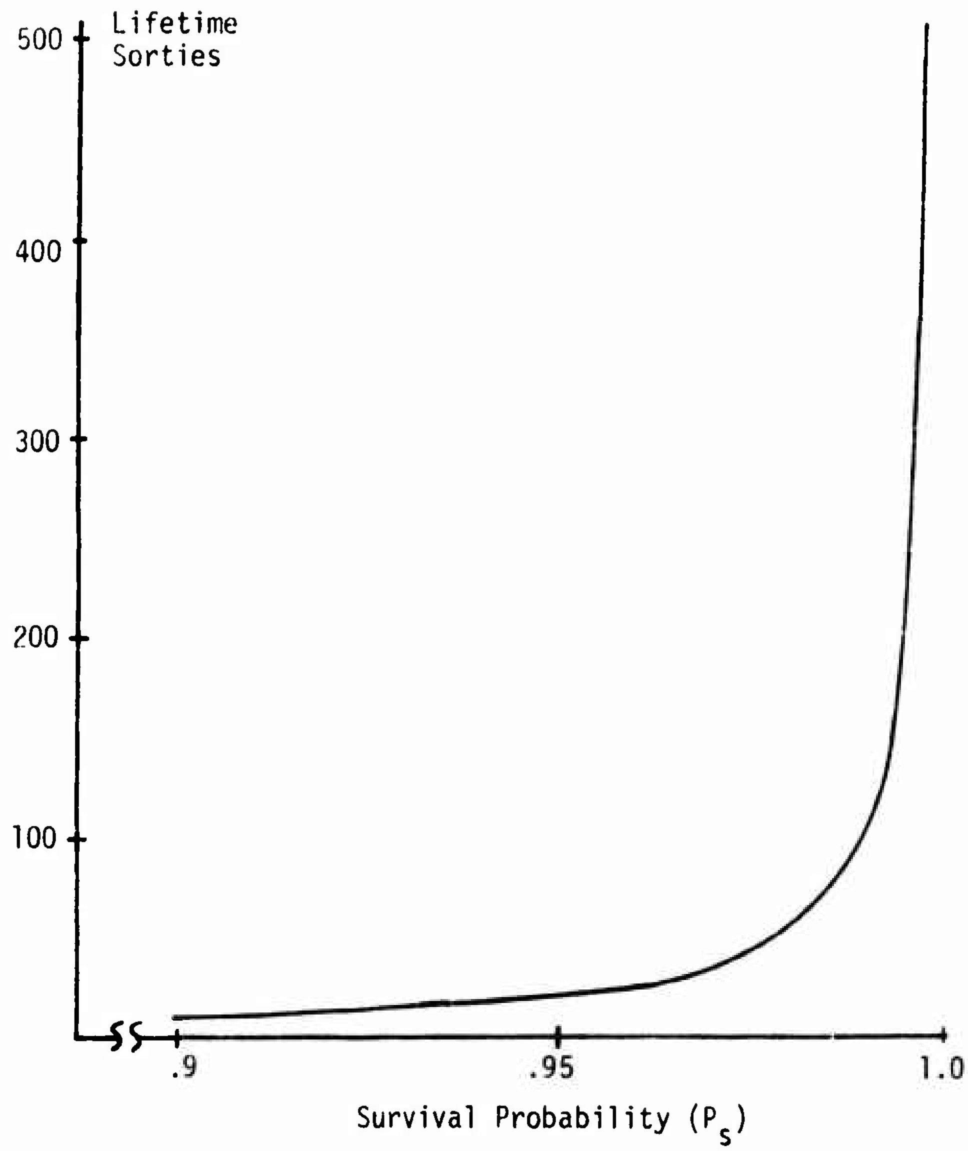


Figure 17. Lifetime Sorties as a Function of Survival Probability.

One obvious means to increase survival probability is to reduce the enemy's defenses (gain air superiority). Survival probability can also be improved by designing the aircraft to reduce the probability of hit (e.g., ECM or reducing radar cross section and IR signature) and to reduce the probability of kill given the aircraft is hit (e.g., armor, foam in fuel tanks). As shown above, any improvements in aircraft survivability significantly enhances mission effectiveness.

b. Comparing Aircraft.

Table XV shows the effectiveness parameters associated with 5 hypothetical aircraft labeled A, B, C, D, and E. Although each parameter is important in the evaluation of an aircraft, it appears impossible to rank the 5 aircraft by studying the table. The table does show that aircraft A has the best kill potential, B has the highest probability of reaching the target without an abort, C has the highest survival probability, and D has the highest availability.

Table XV

EFFECTIVENESS PARAMETERS FOR FIVE AIRCRAFT

EFFECTIVENESS PARAMETERS	AIRCRAFT TYPE				
	A	B	C	D	E
Kill Potential (e.g., targets killed per successful sortie)	2.5	1.8	.80	2.0	2.1
P_c (Probability of reaching target without an abort)	.90	.93	.90	.82	.85
P_s (Survival Probability)	.970	.990	.999	.980	.995
A (Availability)	.85	.87	.83	.90	.83
Mission Time (hr)	2	2	2	2	2
Service Time (hr)	.5	.5	.5	.5	.5

Using the parameters listed in Table XV together with equations (VI-4) and (VI-16), the expected number of targets destroyed as a function of time can be calculated for each aircraft. The results in Figure 18 are based upon continuous operation, i.e., aircraft is launched as soon as it is ready. Although aircraft E does not dominate the others in any of the effectiveness parameters, when all parameters are integrated aircraft E is superior to the others (E and A are about equal in the beginning) at least for time less than 50 hours (about 25 missions). The lifetime targets destroyed by each aircraft indicates where the curves finally level off. The lifetime targets destroyed (LTD) by each aircraft are:

A: LTD = 73

B: LTD = 166

C: LTD = 719

D: LTD = 80

E: LTD = 355

This indicates that C might be better than E since its curve will eventually rise above the targets destroyed curve of aircraft E. Figure 19 shows targets destroyed by C, E, and B as a function of time when time is carried out to 2500 hours (about 859 missions). Although aircraft C and E are the only two competitors, aircraft B is shown merely to demonstrate that its low survivability causes its curve to level off early at a LTD of 166. Figure 19 shows that E is substantially better than C for times less than 1830 hours (629 missions). For times greater than this the higher survivability of C more than compensates for its lower kill potential and C is better than E. The

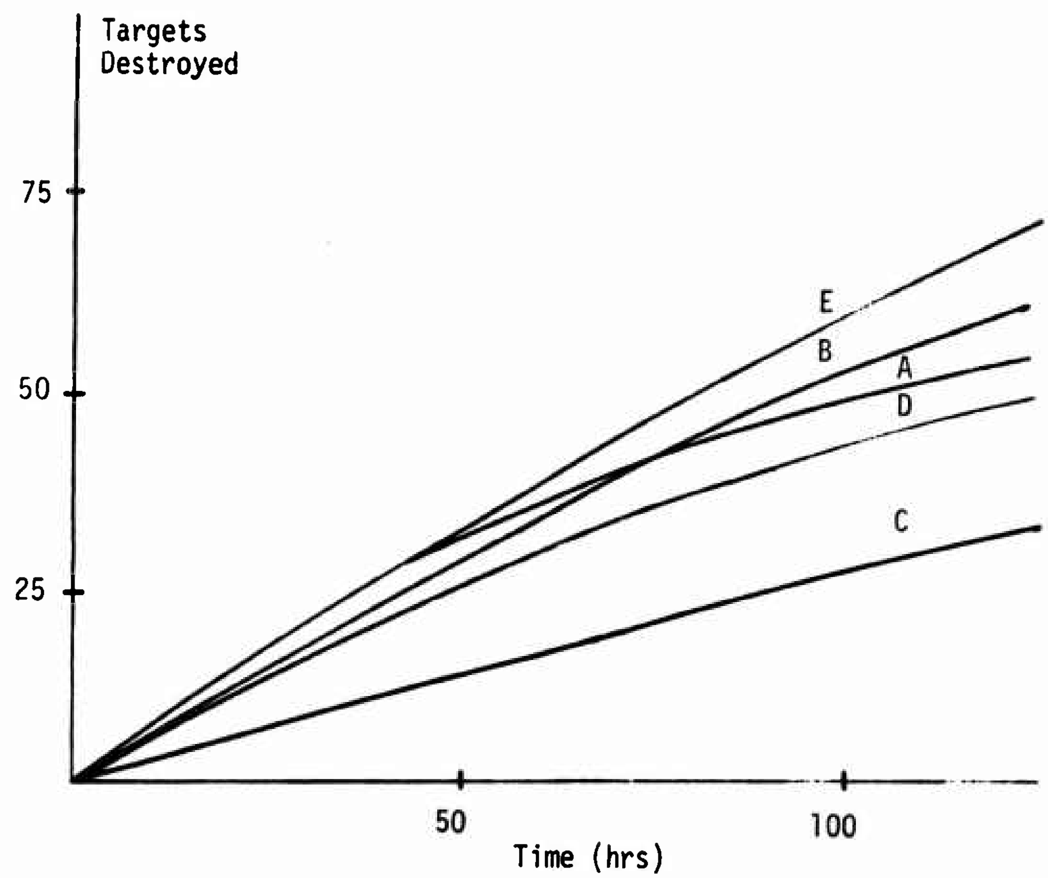


Figure 18. Destruction as a Function of Time for Five Hypothetical Aircraft.

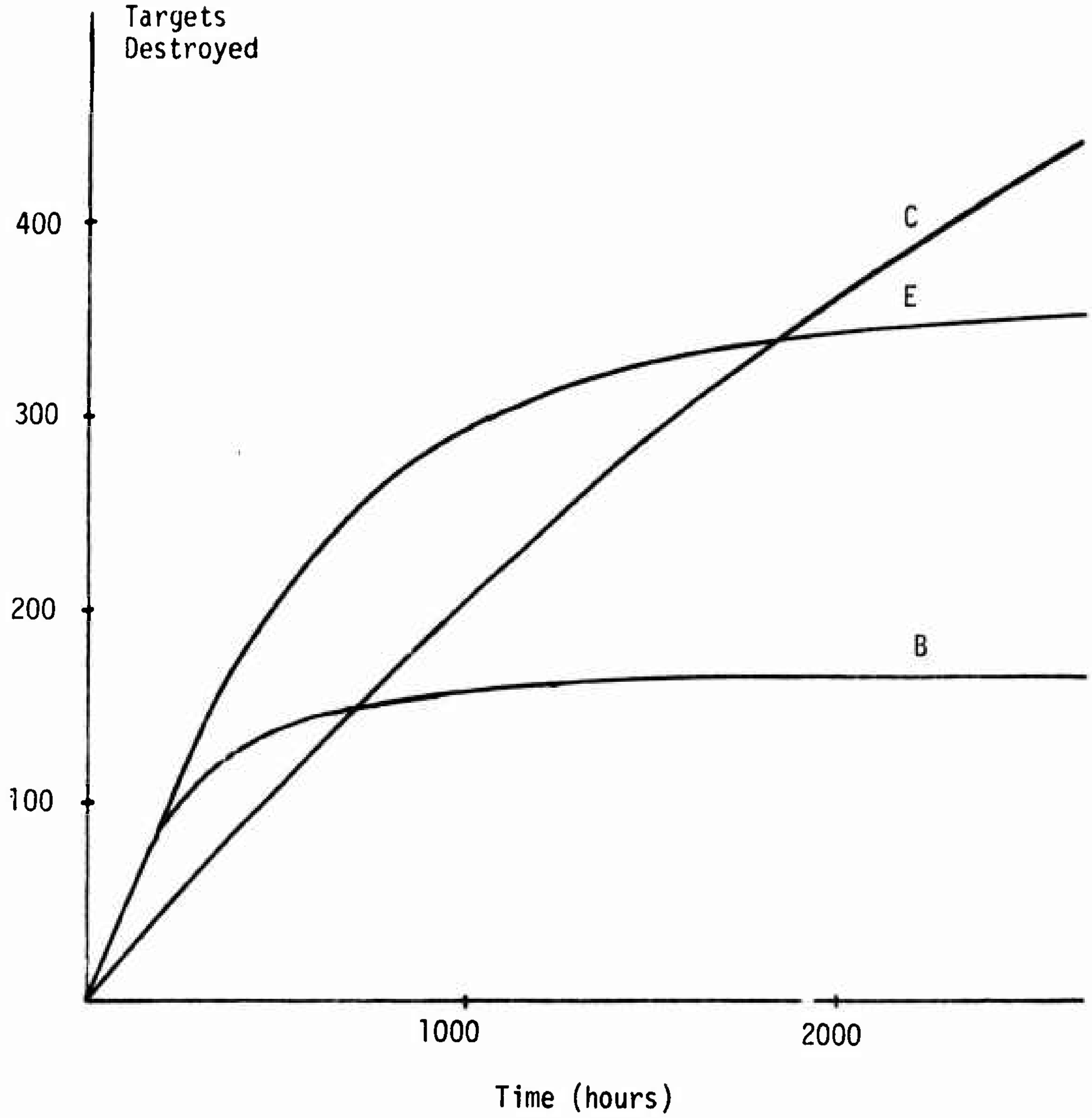


Figure 19. Destruction as a Function of Time for Aircraft B, C, and E.

The analysis shows that E is better than aircraft A, B, and D. However, the selection between C (with a higher LTD) and E is dependent upon the preference of the decision-maker, i.e., whether short term or long term performance is of prime interest.

3. AIR-TO-AIR FIGHTERS (IMPORTANCE OF FIRST SHOT)

It is particularly interesting to apply some of the ideas of the previous section to air-to-air engagements between fighter aircraft. It is intuitively clear that the probability of maneuvering into position to fire the first shot is an important factor in determining the effectiveness of a fighter aircraft. This is due to the fact that the first shot probability has a strong influence on both the kill potential and the survival probability of the fighter. The tools developed in the previous sections provide a means to show quantitatively the influence of first shot probability on the exchange ratio (i.e., Red fighters destroyed per Blue fighter destroyed). This exchange ratio can also be interpreted as the expected number of enemy fighters destroyed during the lifetime of a Blue fighter. The first air-to-air scenario is described in the next paragraph.

In an air-to-air engagement between a Blue and a Red fighter, the probability that the Blue fighter fires the first shot is denoted by P_1. This first shot probability is a function of acquisition and tracking capabilities, speed, maneuverability, and pilot skills. The fighter firing the first shot releases its air-to-air weapons destroying the other fighter with a certain probability (P_{kb} for Blue weapons, P_{kr} for Red weapons). If the attacked fighter is destroyed the engagement is finished; however, for this first scenario it is assumed that if the attacked fighter is not destroyed it maneuvers into position to launch its weapons against the other aircraft (this assumption will be modified later). The engagement is then finished with each fighter getting at most one pass. Although multiple passes could easily be considered, it requires additional assumptions and contributes little to the understanding of the problem (especially if both fighters are assumed to have highly effective air-to-air weapons).

The first quantity to be derived is the probability that the Blue fighter destroys the Red fighter in a given engagement. This is the fighter kill potential; it is equal to the probability that the Blue fighter fires the first shot and destroys the Red fighter plus the probability that the Red fighter fires the first shot and misses the Blue fighter and the Blue fighter then destroys the Red fighter. Thus,

$$\begin{aligned} \rho &= P_1 P_{kb} + (1 - P_1)(1 - P_{kr}) P_{kb} \\ &= P_{kb} \left\{ 1 - P_{kr}(1 - P_1) \right\} , \end{aligned} \qquad \text{(VI-17)}$$

where P_1 denotes the first shot probability of the Blue fighter, P_{kb} is the kill probability of the weapons of the Blue fighter, and P_{kr} is the kill probability of Red weapons.

The next expression to be derived is the single engagement survival probability P_s of the Blue fighter. The Red fighter will be prevented from launching its weapons only if the Blue fighter gets the first shot and destroys the Red fighter. Therefore, the probability that Red attacks the Blue fighter is

$$1 - P_1 P_{kb} \quad . \qquad \text{(VI-18)}$$

The survival probability is then

$$P_s = 1 - P_{kr}(1 - P_1 P_{kb}) \quad . \qquad \text{(VI-19)}$$

From equations (VI-17) and (VI-19) it follows that the exchange ratio is given by

$$ER = \frac{\rho}{1 - P_s} = \frac{P_{kb}\left\{1 - P_{kr}(1 - P_1)\right\}}{P_{kr}\left\{1 - P_1 P_{kb}\right\}} \quad . \qquad \text{(VI-20)}$$

Figure 20 is presented to illustrate the strong influence of first shot probability on both the probability of survival of the Blue fighter and the probability of survival of the Red fighter. In this example the effectiveness of Red and Blue weapons is assumed to be equal, i.e., $P_{kb} = P_{kr} = 0.9$. Although weapons are equally effective, the first shot capability of Blue can cause the survival probability to vary from 0.10 to 0.91 and the kill probability against the Red fighter to vary from 0.09 to 0.90.

Figure 21 incorporates both the kill potential and survival probability to show the dependence of the exchange ratio upon the first shot capability. Two cases are presented corresponding to Red weapon effectiveness of $P_{kr} = 0.6$ and $P_{kr} = 0.9$. For each case the exchange ratio is plotted for $P_{kb} = 0.6$ and $P_{kb} = 0.9$. Several conclusions are apparent:

(1) Effective Blue weapons and a high first shot capability are both necessary for achievement of a high exchange ratio for Blue.

(2) Even when $P_{kb} = 0.9$ and $P_{kr} = 0.6$ a first shot probability below $P_1 = 0.22$ results in an exchange ratio below 1.0, i.e., the advantage of a superior weapon can be nullified by a poor first shot capability.

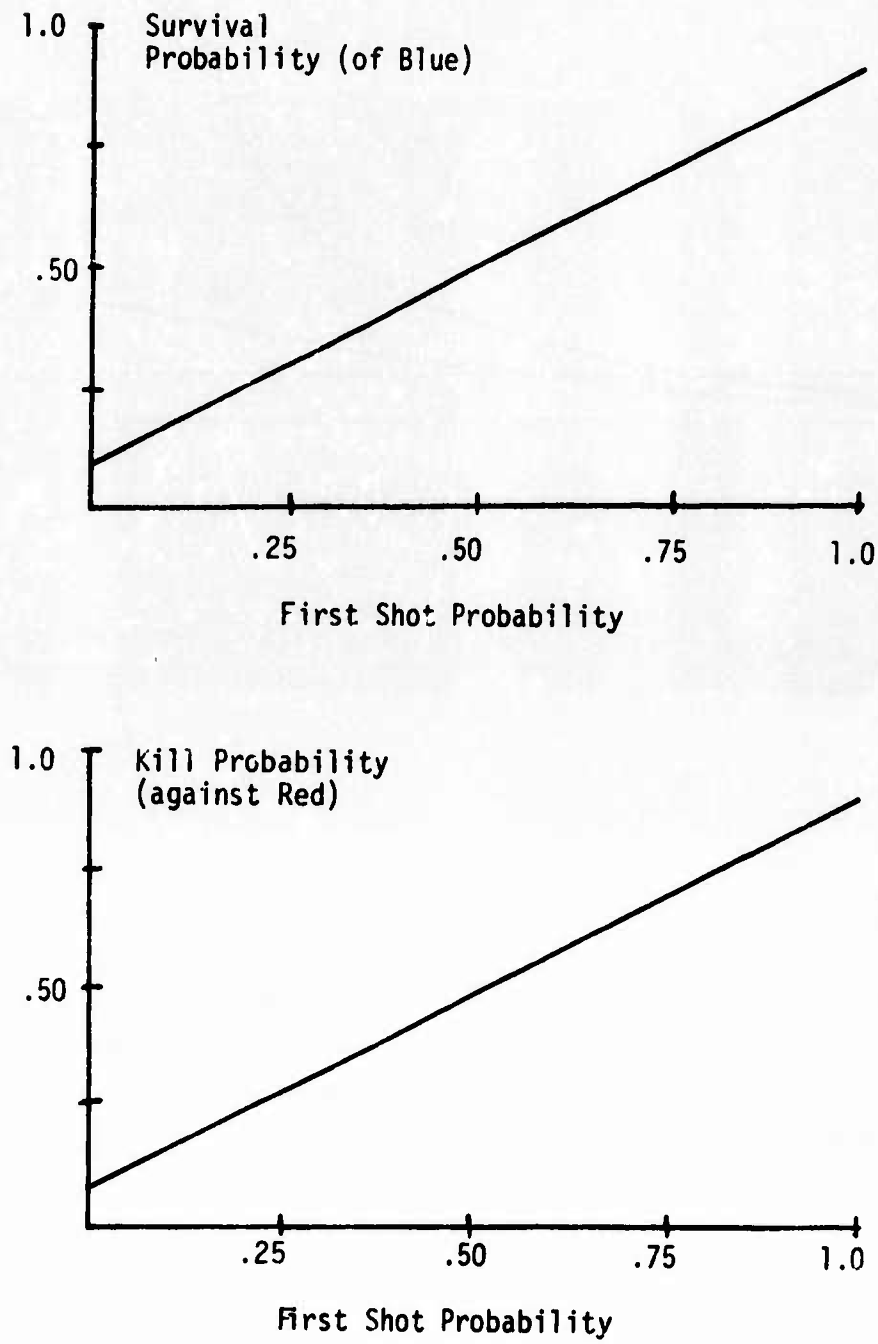


Figure 20. The Effect of First Shot Probability on Survival and Kill Probability ($P_b = P_{kr} = 0.9$).

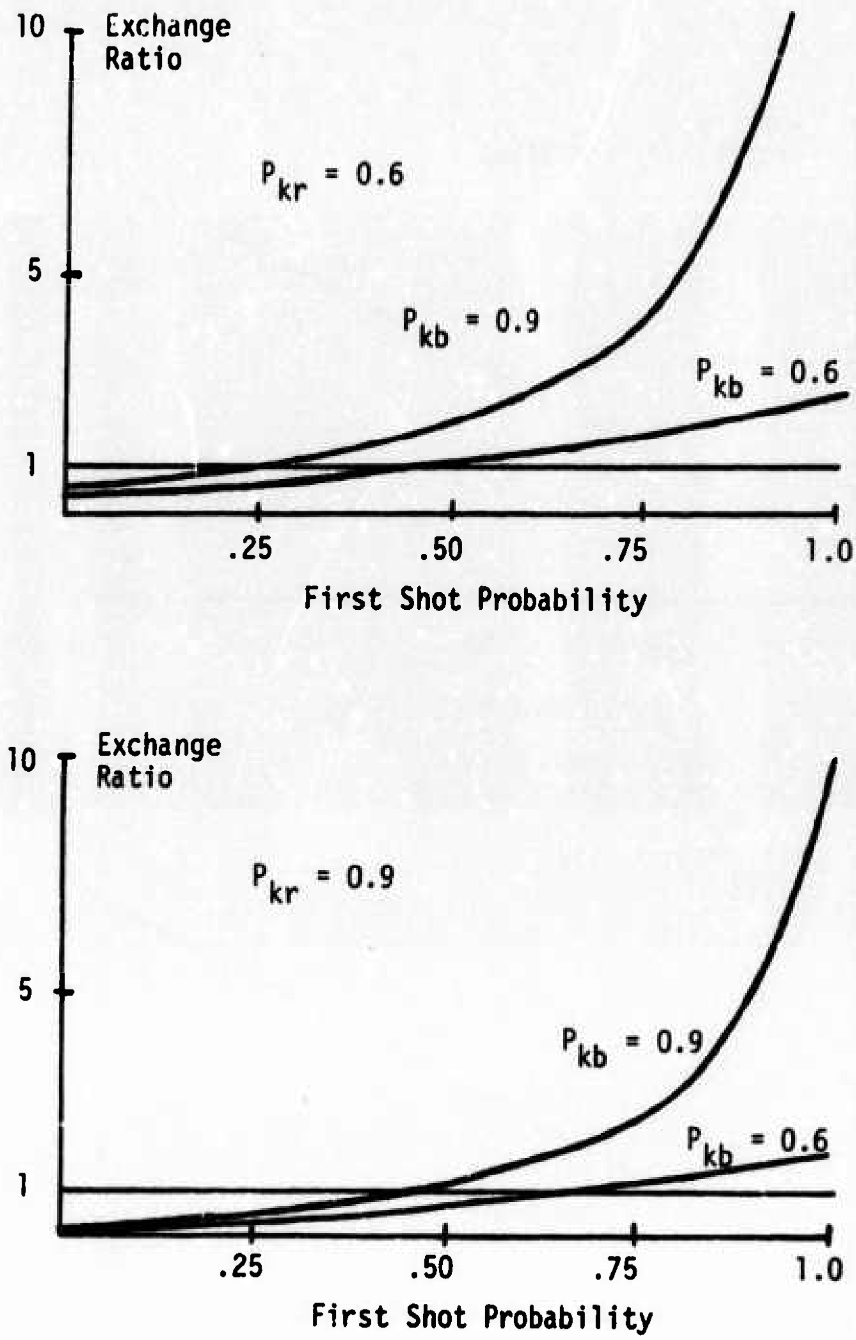


Figure 21. Exchange Ratio as a Function of First Shot Probability.

(3) The disadvantage of a poor weapon (e.g., $P_{kb} = 0.6$ and $P_{kr} = 0.9$) can sometimes be more than compensated for by a high first shot capability.

In the previous scenario it was assumed that whenever the fighter firing the first shot missed, the other fighter then maneuvered into position to fire its weapons. However, a fighter may fire the first shot and miss but still have the capability to outmaneuver the other fighter thereby avoiding being fired upon. To account for this, the following probabilities are introduced:

P_{mb} = Probability Blue fighter avoids being fired upon whenever it fires first shot and misses.

P_{mr} = Probability Red fighter avoids being fired upon whenever it fires first shot and misses.

The probability that the Red fighter is destroyed becomes

$$\rho = P_{kb}\left\{P_1 + (1 - P_1)(1 - P_{kr})(1 - P_{mr})\right\} \quad . \qquad \text{(VI-21)}$$

The probability that the Blue fighter is destroyed is

$$\begin{aligned} 1 - P_s &= P_1(1 - P_{kb})(1 - P_{mb})P_{kr} + (1 - P_1)P_{kr} \\ &= P_{kr}\left\{1 - P_1(P_{kb} + P_{mb} - P_{mb}P_{kb})\right\} \quad . \qquad \text{(VI-22)} \end{aligned}$$

From equations (VI-21) and (VI-22) it follows that the exchange ratio is

$$ER = \frac{P_{kb}\left\{P_1 + (1 - P_1)(1 - P_{kr})(1 - P_{mr})\right\}}{P_{kr}\left\{1 - P_1(P_{kb} + P_{mb} - P_{mb}P_{kb})\right\}} \quad . \qquad \text{(VI-23)}$$

For $P_{mb} = P_{mr} = 0$, equation (VI-23) reduces to equation (VI-20).

The most favorable case for Blue is when $P_{mb} = 1$ and $P_{mr} = 0$; the most unfavorable case is $P_{mb} = 0$ and $P_{mr} = 1$. Using these extreme cases, the bounds for the exchange ratio are shown in Figure 22. The solid curves are identical to those in Figure 21, i.e., $P_{mb} = P_{mr} = 0$. As seen by comparing the solid and lower curves in the Figures, if Red can outmaneuver Blue after getting first shot but Blue does not have this capability ($P_{mb} = 0$, $P_{mr} = 1$) this has little effect on the exchange ratio since it has no effect on Blue's survival probability. However, if $P_{mb} = 1$ and $P_{mr} = 0$, Blue can improve its survival probability, and hence the exchange ratio is improved significantly if the first shot probability is high; furthermore, the lower the value of P_{kb} the greater the importance of the capability of Blue being able to outmaneuver Red after firing the first shot.

4. CONCLUSIONS

(1) An evaluation of the effectiveness of an aircraft must account for the interaction of availability, abort probability, kill potential, and survivability. Individually, these characteristic parameters do not determine the worth of an aircraft.

(2) Any valid measure of effectiveness must also account for the cumulative effect of repeated sorties.

(3) The measures of effectiveness developed here provide a simple means of integrating the characteristic effectiveness parameters to determine the cumulative damage accrued by repeated sorties.

(4) Survival probability can be the most dominant factor in determining the lifetime effectiveness of an aircraft. For example, a 5% increase in kill potential results in a 5% increase in lifetime damage; however, a 5% increase in survival probability, say from $P_s = .95$ to $P_s = .9975$, results in a 2100% increase in targets destroyed during the lifetime of the aircraft.

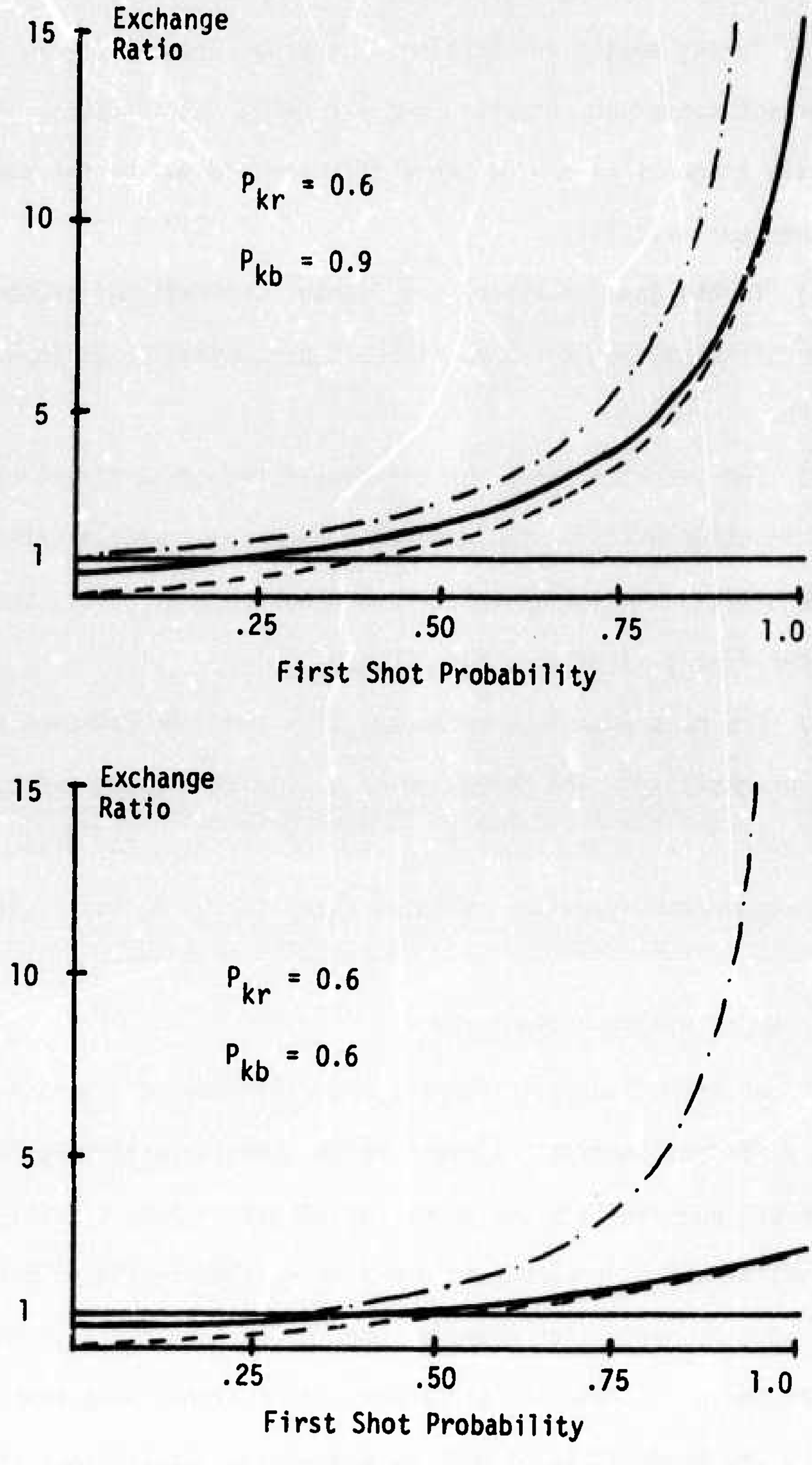


Figure 22. Importance of Maneuverability after First Shot (P_{kr} = 0.6).

(5) Since survivability is of such great importance it warrants special emphasis during design and testing. Survival probability is an extremely important factor in comparing two aircraft; for instance, one aircraft may have a poorer weapon delivery accuracy and yet be far superior because of higher survivability.

(6) In the case of air-to-air fighter aircraft the exchange ratio (Red aircraft destroyed per Blue aircraft destroyed) is an important measure of worth.

(7) The exchange ratio for air-to-air fighter aircraft can be expressed as a function of three fundamental parameters: weapon effectiveness, first shot probability, and the capability to maneuver away (avoid being fired upon) after firing first shot and missing.

(8) The most important parameter affecting the exchange ratio is the first shot probability. The advantage of a superior weapon can be nullified by a poor first shot capability; and, conversely, the disadvantage of an inferior weapon can sometimes be compensated for by a good first shot capability.

5. MULTI-ROLE, MULTI-MISSION CAPABILITY

The measures of effectiveness discussed above provide very useful tools for evaluating different candidate systems to be used in performing the same missions. However, when the systems under consideration have a multi-role or mission capability, a more elaborate means of evaluating the effectiveness of each system must be used. For example, when evaluating two systems which can perform both the air-to-air and air-to-ground missions, some means must be devised to allow the tradeoff to be made by evaluating the system's effectiveness under both roles. Unless one system clearly dominates the other in both roles, some measure of effectiveness must be used which allows their effectiveness to be evaluated in conjunction with their ability to complement the existing base force and its capabilities in performing the air-to-air and air-to-ground missions. That is, the current base force may be weak in the air-to-air capability and thus the addition of the superior air-to-air system would be preferred. On the other hand, by adding a better air-to-ground system to a force which lacks this capability, it may be possible to allow better utilization of an existing air-to-air capability which was previously needed in the air-to-ground role.

6. SUMMARY

Two measures of effectiveness for fighter aircraft have been presented. In the case of air-to-ground fighters, it was shown that an evaluation of the effectiveness must account for the interaction of availability, abort probability, kill potential, and survivability; and survivability is often the most dominant factor. For air-to-air fighters the exchange ratio (Red aircraft destroyed per Blue aircraft destroyed) is an important measure of worth, and it can be expressed as a function of weapon effectiveness, maneuver capability, and first shot probability with first shot probability being the most important parameter.

SECTION VII

SUMMARY

In summary, Figure 23 presents a progression of the new and innovative approach for obtaining higher operational reliability levels using the models and methodology developed in this study.

Each block represents a necessary step in the process, and continuous feedback and iteration is required to realize the full potential of the approach. The feedback loop extends from any one block to any preceding block. By establishing this sequence with the appropriate feedback and iteration, requirements and achievable operational levels can be kept compatible.

Starting with the requirements and proceeding to reliability testing must involve a great deal more than MIL-STD-781B demonstrations. If Initial Operational Test and Evaluation (IOT&E) results are not available, then the equipment should be stressed in the laboratory in such a way as to uncover as many reliability deficiencies as possible. If test results indicate that the equipment in its original configuration will not meet operational requirements, then the requirements can either be adjusted or reliability improvement programs can be undertaken. The MCSP model is then used to evaluate the original configuration by identifying the critical components and determining the effect of critical component improvement on overall system reliability. The next step is to determine realistic funding levels for the reliability improvement program. This is accomplished with a reliability management program in which reliability options and logistic support costs are considered.

With this data the DSPC model can be implemented. This methodology systematically identifies those subsystem options which provide the highest system performance at any prescribed level of cost. Along with the DSPC methodology appropriate

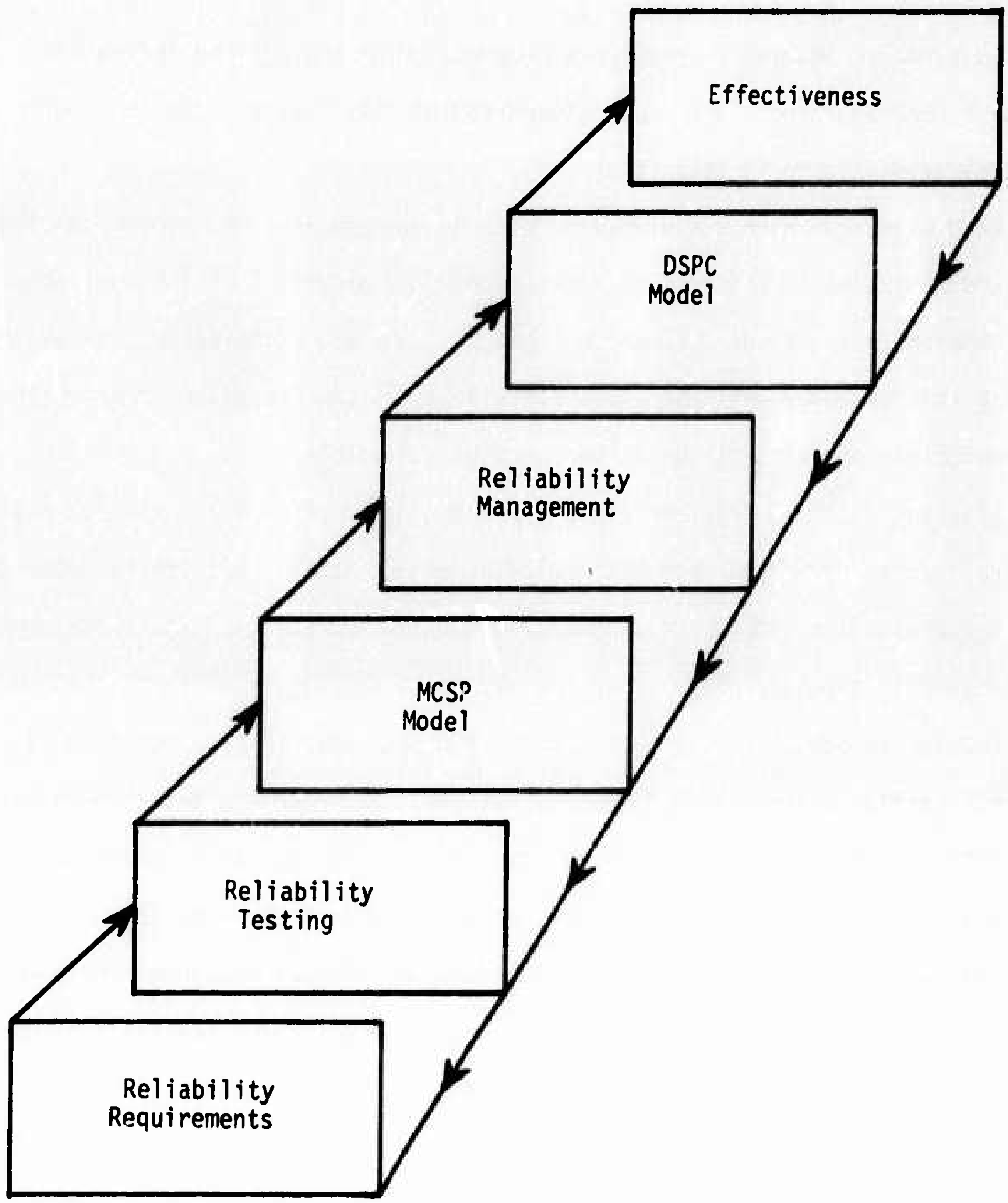


Figure 23. Implementing the Methodology to Achieve Higher Operational Reliability Levels.

measures of effectiveness must be tailored to the particular mission of interest and related to the system performance parameters.

Two measures of effectiveness for fighter aircraft have been presented. In the case of air-to-ground fighters, it was shown that an evaluation of the effectiveness must account for the interaction of availability, abort probability, kill potential, and survivability; and survivability is often the most dominant factor. For air-to-air fighters, the exchange ratio (Red aircraft destroyed per Blue aircraft destroyed) is an important measure of worth, and it can be expressed as a function of weapon effectiveness, maneuver capability, and first shot probability with first shot probability being the most important parameter.

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APPENDIX A

MISSION COMPLETION SUCCESS PROBABILITY (MCSP) COMPUTER PROGRAM

1. DEFINITIONS OF INPUTS

NSSYS ≡ Total number of subsystems (NSSYS $\leq$ 50). Each subsystem is identified by number and name.

NPHASES ≡ Number of mission phases (NPHASES $\leq$ 20).

TO(i, j) ≡ Operating Time of i-th subsystem during j-th mission phase (i = 1, 2, ..., NSSYS; j = 1, 2, ..., NPHASES).

PA(i, j) ≡ Conditional probability of mission abort given that the i-th subsystem has a failure during the j-th mission phase (i = 1, 2, ..., NSSYS; j = 1, 2, ..., NPHASES).

IND1(i) ≡ Indicator equal to 0 or 1. If IND1(i) = 0 then the baseline for the i-th subsystem is nonredundant. If IND1(i) = 1 then the baseline for the i-th subsystem is redundant.

F(i, 1) ≡ When IND1(i) = 0, this parameter is required to denote the mean operating time between failure of subsystem i.

NR(i, 1) ≡ When IND1(i) = 1, this parameter is required to denote the number of redundant units for subsystem i (NR(i, 1) $\leq$ 5).

R(i, 1) ≡ When IND1(i) = 1, this parameter is defined as 0 or 1. If R(i, 1) = 0 then the redundancy for subsystem i is operative. If R(i, 1) = 1 then the redundancy is standby.

FR(i, 1, k) ≡ When IND1(i) = 1, this parameter denotes the mean operating time between failure of the k-th redundant subsystem of subsystem i (k = 1, 2, ..., NR(i, 1)).

IND 2 ≡ Indicator equal to 0 or 1. If IND 2 = 1 then the subsystems will be ranked according to their probability of causing an abort.

IND 3 ≡ Indicator equal to 0 or 1. If IND 3 = 1 then a sensitivity analysis will be performed for improvement of a selected nonredundant subsystem.

IS ≡ When IND 3 = 1, this identifies the number of the nonredundant subsystem whose MTBF is to be incremented.

DELTA ≡ When IND 3 = 1, this denotes the size of the MTBF increment for subsystem S.

XLIMIT ≡ When IND 3 = 1, this denotes the upper limit of the MTBF increment for subsystem S.

2. OUTPUTS OF THE MCSP MODEL

The model calculates the probability of mission completion without an abort causing failure of a subsystem. If IND 2 = 1 then the subsystems will be ranked according to the probability of an abort causing failure of each subsystem. If IND 3 = 1 then the mean operating time between failure of one nonredundant subsystem will be incremented and the corresponding values of MCSP will be calculated. To perform a sensitivity analysis on a redundant subsystem the model could be exercised repeatedly making the appropriate changes in the F(i, 1, k) for each case. It should be mentioned that in the case of standby redundancy of a subsystem the corresponding MTBFs (FR(i, 1, k), k = 1, 2, ..., NR(i, 1)) must be input as either all equal or all unequal.

3. MCSP CARD INPUTS

The card inputs to the MCSP model are identical to those for the DSPC model (described in Appendix B) with a few exceptions.

There are fewer arrays to be input, with several having a constant rather than variable dimension. The same DSPC rules for array input hold for these arrays with the appropriate dimension set to 1.

There is an extra "array" to be input to this program. The mnemonic is IND and the fields are:

FIELD	VALUE OF
1	IND 2
2	IND 3
3	IS
4	DELTA
5	XLIMIT

4 - MCSP PROGRAM LISTING

PROGRAM MCSP(INPUT,OUTPUT,TAPE60=INPUT)

```
      PROGRAM MCSP(INPUT,OUTPUT,TAPE60=INPUT)
C
C     INPUT THE ARRAYS
C
      CALL INPUT
C
C     NOW EXECUTE THE MAIN BODY
C
      CALL CONTROL
C
C     THATS ALL FOLKS....
      END
```

```
      SUBROUTINE INPUT
      COMMON NSSYS,NPHASES,IND2,IND3,IS,DELTA,XLIMIT,TO(50,20),PA(50,20)
     1,NAMES(50),IND1(50),NR(50,1),R(50,1),F(50,1),FR(50,1,6),TP(50),P(5
     20,1),PMC(50)
      DIMENSION CARDS(8)
      IABORT=0
C
C     READ THE FIRST CARD
C
      READ 19, CARDS
      NSSYS=CARDS(2)
      NPHASES=CARDS(3)
      PRINT 20, NSSYS,NPHASES
      READ 21, (NAMES(I),I=1,NSSYS)
      PRINT 22, (NAMES(I),I=1,NSSYS)
C
C     BEGIN READING THE ARRAYS
C
1     READ 23, IAN
      IF (EOF(60)) 18,2
2     PRINT 24, IAN
      IF (IAN.EQ.4HTO  ) GO TO 3
      IF (IAN.EQ.4HPA  ) GO TO 5
      IF (IAN.EQ.4HIND1) GO TO 7
      IF (IAN.EQ.4HIND ) GO TO 9
      IF (IAN.EQ.4HNR  ) GO TO 10
      IF (IAN.EQ.4HR   ) GO TO 12
      IF (IAN.EQ.4HFR  ) GO TO 14
      IF (IAN.EQ.4HF   ) GO TO 16
C     ERROR ON CARD
      PRINT 25
      IABORT=1
      GO TO 1
C
C     TO       OPERATING TIME
C
3     DO 4 I=1,NSSYS
      DO 4 J=1,NPHASES,8
      PRINT 27, I,J
      CALL READCD (CARDS)
      DO 4 L=1,8
      K=L+J-1
      IF (K.GT.NPHASES) GO TO 4
      TO(I,K)=CARDS(L)
4     CONTINUE
      GO TO 1
C
C     PA       PROBABILITY OF MISSION ABORT
C
5     DO 6 I=1,NSSYS
```

```
      DO 6 J=1,NPHASES,8
      PRINT 28, I,J
      CALL READCD (CARDS)
      DO 6 L=1,8
      K=L+J-1
      IF (K.GT.NPHASES) GO TO 6
      PA(I,K)=CARDS(L)
6     CONTINUE
      GO TO 1
C
C     IND1     REDUNDANT SUBSYSTEM INDICATOR
C
7     DO 8 I=1,NSSYS,8
      PRINT 29, I
      CALL READCD (CARDS)
      DO 8 J=1,8
      K=I+J-1
      IF (K.GT.NSSYS) GO TO 8
      IND1(K)=CARDS(J)
8     CONTINUE
      GO TO 1
C
C     IND      INDICATORS OF PROGRAM OPTIONS
C
9     CALL READCD1 (CARDS)
      IND2=CARDS(1)
      IND3=CARDS(2)
      IS=CARDS(3)
      DELTA=CARDS(4)
      XLIMIT=CARDS(5)
      PRINT 26, IND2,IND3,IS,DELTA,XLIMIT
      GO TO 1
C
C     NR       NUMBER OF REDUNDANT SUBSYSTEMS /SUBSYSTEM
C
10    DO 11 I=1,NSSYS
      PRINT 30, I
      CALL READCD (CARDS)
      NR(I,1)=CARDS(1)
11    CONTINUE
      GO TO 1
C
C     R        OPERATIVE OR STANDBY REDUNDANCY
C
12    DO 13 I=1,NSSYS
      PRINT 31, I
      CALL READCD (CARDS)
      R(I,1)=CARDS(1)
13    CONTINUE
      GO TO 1
```

```
C
C     FR       MTBF FOR REDUNDANT SUBSYSTEMS
C
14    DO 15 I=1,NSSYS
      NRO=NR(I,1)
      PRINT 32, I
      CALL READCD (CARDS)
      DO 15 J=1,NRO
      FR(I,1,J)=CARDS(J)
15    CONTINUE
      GO TO 1
C
C     F        MTBF FOR NON-REDUNDANT SUBSYSTEMS
C
16    DO 17 I=1,NSSYS
      PRINT 33, I
      CALL READCD (CARDS)
      F(I,1)=CARDS(1)
17    CONTINUE
      GO TO 1
18    IF (IABORT.EQ.1) CALL EXIT
      RETURN
C
C
19    FORMAT (8F10.0)
20    FORMAT (1H1,5X,23HNUMBER OF SUBSYSTEMS = ,I5,5X,26HNUMBER OF PHASE
     1S/MISSION =,I5)
21    FORMAT (8A10)
22    FORMAT (/,17H  SUBSYSTEM NAMES/,5(10(3X,A10)/))
23    FORMAT (A4)
24    FORMAT (/,17H  NEW ARRAY, ID= ,A4)
25    FORMAT (/88H ERROR ON ARRAY TYPE CARD, WILL CONTINUE READING PARAM
     1ETER DECK BEFORE ABORTING THE JOB.)
26    FORMAT (8H IND2 = ,I2,9H ,IND3 = ,I2,14H ,SUBSYSTEM = ,I2,14H ,INC
     1REMENT = ,F6.0,12H ,MAXIMUM = ,F6.0)
27    FORMAT (4H TO(,I2,1H,,I2,4H) = )
28    FORMAT (4H PA(,I2,1H,,I2,4H) = )
29    FORMAT (6H IND1(,I2,4H) = )
30    FORMAT (4H NP(,I2,6H,1) = )
31    FORMAT (3H R(,I2,6H,1) = )
32    FORMAT (4H FR(,I2,8H,1,1) = )
33    FORMAT (3H F(,I2,6H,1) = )
      END
```

SUBROUTINE READCD (CARDS)

```
      SUBROUTINE READCD (CARDS)
      DIMENSION CARDS(8), ICARD(8)
      DATA (IB=10H          )
      NOP=0
      GO TO 1
      ENTRY READCD1
      NOP=1
1     READ 6, ICARD
      IF (EOF(60)) 2,3
2     PRINT 7
      CALL EXIT
C     CHECK TO SEE WHICH IS THE LAST NON-BLANK WORD
3     DO 4 I=1,8
      IF (ICARD(I).EQ.IB) GO TO 4
      IT=I
4     CARDS(I)=0.
      NC=IT*10
      DECODE (NC,8,ICARD) (CARDS(I),I=1,IT)
      IF (NOP.EQ.1) GO TO 5
      PRINT 9, (CARDS(I),I=1,IT)
5     CONTINUE
      RETURN
C
C
6     FORMAT (8A10)
7     FORMAT (//,61H  END-OF-FILE READ INSTEAD OF PARAMETER CARD. JOB AB
     1ORTED....)
8     FORMAT (8F10.2)
9     FORMAT (1H+,20X,8F10.2)
      END
```

FUNCTION F1 (I,J)

```
      FUNCTION F1 (I,J)
      COMMON NSSYS,NPHASES,IND2,IND3,IS,DELTA,XLIMIT,TO(50,20),PA(50,20)
     1,NAMES(50),IND1(50),NR(50,1),R(50,1),F(50,1),FR(50,1,6),TP(50),P(5
     20,1),PMC(50)
      F1=EXP(-TP(I)/F(I,J))
      RETURN
      END
```

FUNCTION F2 (I,J)

```
      FUNCTION F2 (I,J)
      COMMON NSSYS,NPHASES,IND2,IND3,IS,DELTA,XLIMIT,TO(50,20),PA(50,20)
     1,NAMES(50),IND1(50),NR(50,1),R(50,1),F(50,1),FR(50,1,6),TP(50),P(5
     20,1),PMC(50)
      TEMP=1.
      NRO=NR(I,J)
      DO 1 L=1,NRO
    1 TEMP=TEMP*(1.-EXP(-TP(I)/FR(I,J,L)))
      F2=1.-TEMP
      RETURN
      END
```

FUNCTION F3 (I,J)

```
      FUNCTION F3 (I,J)
      COMMON NSSYS,NPHASES,IND2,IND3,IS,DELTA,XLIMIT,TO(50,20),PA(50,20)
     1,NAMES(50),IND1(50),NR(50,1),R(50,1),F(50,1),FR(50,1,6),TP(50),P(5
     20,1),RMC(50)
      TEMP=TP(I)/FR(I,J,1)
      T1=EXP(-TEMP)
      SUM=0.
      NRO=NR(I,J)
      DO 1 L=1,NRO
    1 SUM=SUM+((TEMP)**(L-1))/IFAC(L-1)
      F3=T1*SUM
      RETURN
      END
```

FUNCTION IFAC (I)

```
      FUNCTION IFAC (I)
C     COMPUTES THE FACTORIAL
      ITEMP=1
      IF (I.EQ.0) 1,2
1     IFAC=1
      RETURN
2     DO 3 K=1,I
3     ITEMP=ITEMP*K
      IFAC=ITEMP
      RETURN
      END
```

FUNCTION G5 (I, J,M,XX)

```
      FUNCTION G5 (I,J,M,XX)
      COMMON NSSYS,NPHASES,IND2,IND3,IS,DELTA,XLIMIT,TO(50,20),PA(50,20)
     1,NAMES(50),IND1(50),NR(50,1),R(50,1),F(50,1),FR(50,1,6),TP(50),P(5
     20,1),PMC(50)
      IF (M.EQ.1) GO TO 3
      SUM=0.
      DO 2 L=1,M
      SUM1=1.
      DO 1 K=1,M
      IF (K.EQ.L) GO TO 1
      SUM1=SUM1*(FR(I,J,L)-FR(I,J,K))
 1    CONTINUE
 2    SUM=SUM+FR(I,J,L)**(M-1)*EXP(-XX/FR(I,J,L))/SUM1
      G5=SUM
      RETURN
 3    G5=EXP(-XX/FR(I,J,1))
      RETURN
      END
```

FUNCTION F5 (I,J)

```
      FUNCTION F5 (I,J)
      COMMON NSSYS,NPHASES,IND2,IND3,IS,DELTA,XLIMIT,TO(50,20),PA(50,20)
     1,NAMES(50),IND1(50),NR(50,1),R(50,1),F(50,1),FR(50,1,6),TP(50),P(5
     20,1),PMC(50)
      M=NR(I,J)
      XX=TP(I)
      F5=G5(I,J,M,XX)
      RETURN
      END
```

SUBROUTINE TSORT (A,N)

```
      SUBROUTINE TSORT (A,N)
      DIMENSION A(1), IL(16), IU(16)
      I=1
      J=N
      M=0
 1    IF (J.LE.I) GO TO 9
 2    IJ=(I+J)/2
      K=I
      L=J
      IF (A(I).LE.A(J)) GO TO 3
      T=A(J)
      A(J)=A(I)
      A(I)=T
 3    T=A(IJ)
      IF (A(I).LE.T) GO TO 4
      A(IJ)=A(I)
      A(I)=T
      T=A(IJ)
      GO TO 5
 4    IF (T.LE.A(J)) GO TO 5
      A(IJ)=A(J)
      A(J)=T
      T=A(IJ)
 5    L=L-1
      IF (T.LT.A(L)) GO TO 5
      TT=A(L)
 6    K=K+1
      IF (A(K).LT.T) GO TO 6
      IF (L.LT.K) GO TO 7
      A(L)=A(K)
      A(K)=TT
      GO TO 5
 7    M=M+1
      IF (L-I.LE.J-K) GO TO 8
      IL(M)=I
      IU(M)=L
      I=K
      GO TO 10
 8    IL(M)=K
      IU(M)=J
      J=L
      GO TO 10
 9    IF (M.EQ.0) RETURN
      I=IL(M)
      J=IU(M)
      M=M-1
 10   IF (J-I.GE.13) GO TO 2
      IF (I.EQ.1) GO TO 1
 11   I=I+1
      IF (J.LT.I) GO TO 9
```

SUBROUTINE TSORT (A,N)

```
      T=A(I)
      IF (A(I-1).LE.T) GO TO 11
      K=I-1
12    A(K+1)=A(K)
      K=K-1
      IF (T.LT.A(K)) GO TO 12
      A(K+1)=T
      GO TO 11
      END
```

```
      SUBROUTINE CONTROL
      COMMON NSSYS,NPHASES,IND2,IND3,IS,DELTA,XLIMIT,TO(50,20),PA(50,20)
     1,NAMES(50),IND1(50),NR(50,1),R(50,1),F(50,1),FR(50,1,6),TP(50),P(5
     20,1),PMC(50)
      DIMENSION XTEMP(50), ITEMP(50)
C
C     PART I.      CALCULATE P
C
      DO 5 I=1,NSSYS
      TP(I)=0.
      DO 1 J=1,NPHASES
    1 TP(I)=TP(I)+TO(I,J)*PA(I,J)
      IF (IND1(I).EQ.1) GO TO 2
C
C     NON-REDUNDANT SUBSYSTEM
C
      P(I,1)=F1(I,1)
      GO TO 5
    2 IF (R(I,1).EQ.1.) GO TO 3
C
C     OPERATIVE REDUNDANCY
C
      P(I,1)=F2(I,1)
      GO TO 5
C
C     STANDBY REDUNDANCY
C
    3 IF (FR(I,1,1).NE.FR(I,1,2)) GO TO 4
C
C     EQUAL MTBF
C
      P(I,1)=F3(I,1)
      GO TO 5
C
C     UNEQUAL MTBF
C
    4 P(I,1)=F5(I,1)
    5 CONTINUE
C
C     PART II.     MCSP
C
      PMC(1)=1.
      DO 6 I=1,NSSYS
    6 PMC(1)=PMC(1)*P(I,1)
C
C     PART III.    RANK ABORT CAUSING SUBSYSTEMS
C
      DO 7 I=1,NSSYS
    7 XTEMP(I)=P(I,1)
      CALL TSORT (XTEMP,NSSYS)
```

```
      DO 8 I=1,NSSYS
      DO 8 J=1,NSSYS
      IF (XTEMP(I).NE.P(J,1)) GO TO 8
      ITEMP(I)=J
8     CONTINUE
      DO 9 I=1,NSSYS
9     P(I,1)=1-XTEMP(I)
      PRINT 15, PMC(1)
      IF (IND2.EQ.0) GO TO 11
      PRINT 16
      DO 10 I=1,NSSYS
      J=ITEMP(I)
      PRINT 17, I,NAMES(J),J,P(I,1)
10    CONTINUE
C
C     PART IV.     SESITIVITY ANALYSIS OF NON-REDUNDANT SUBSYSTEM, IS
11    IF (IND3.EQ.0) GO TO 14
      MAX=XLIMIT/DELTA+1.
      FS=F(IS,1)
      TPS=TP(IS)
      DO 12 I=2,MAX
      DELT=(I-1)*DELTA
12    PMC(I)=PMC(1)*EXP(-TPS*(1./(FS+DELT)-1./FS))
      PRINT 18, NAMES(IS),FS
      DO 13 I=1,MAX
      DELT=(I-1)*DELTA
      PRINT 19, DELT,PMC(I)
13    CONTINUE
14    CONTINUE
      RETURN
C
C
15    FORMAT (1H1,17X,7HMCSP = ,F8.6)
16    FORMAT (//,12X,9HSUBSYSTEM,5X,9HSUBSYSTEM,5X,11HPROBABILITY,/,3X,4
     1HRANK,7X,4HNAME,9X,6HNUMBER,9X,8HOF ABORT)
17    FORMAT (4X,I2,6X,A10,6X,I2,11X,F8.6)
18    FORMAT (///,2X,32HNAME OF SUBSYSTEM INCREMENTED = ,A10,/,5X,22HINI
     1TIAL MTBF(HOURS) = ,F10.2//,2X,9HINCREMENT,6X,4HMCSP)
19    FORMAT (2X,F8.1,5X,F8.6)
      END
```

APPENDIX B

DESIGNING TO SYSTEM PERFORMANCE/COST (DSPC) COMPUTER PROGRAM

1. DEFINITIONS OF INPUTS

NSYS ≡ Total number of systems (e.g., fleet size).

NSSYS ≡ Total number of subsystems (NSSYS $\leq$ 40). Each subsystem is identified by number and name.

NPHASES ≡ Number of mission phases (NPHASES $\leq$ 20).

LASTP ≡ Phase through which MCSP is to be calculated (this is usually NPHASES or the target phase).

NYEARS ≡ Number of years to be considered in calculating logistic support costs (e.g., system lifetime).

NMPM ≡ Average number of missions per month per system.

TR(i) ≡ Ratio of total operating time to mission operating time of the i-th subsystem.

TO(i, j) ≡ Operating time of the i-th subsystem during the j-th mission phase (i = 1, 2, ..., NSSYS; j = 1, 2, ..., NPHASES).

PA(i, j) ≡ Conditional probability of mission abort given that the i-th subsystem has a failure during the j-th mission phase.

N(i) ≡ Number of nonredundant options (other than the baseline subsystem) for the i-th subsystem (N(i) $\leq$ 5).

RO(i) ≡ Number of redundancy options (other than the baseline) for the i-th subsystem (RO(i) $\leq$ 5).

IND1(i) ≡ Indicator equal to 0 or 1. If IND1(i) = 0 then the baseline for the i-th subsystem is nonredundant. If IND1(i) = 1 then the baseline for the i-th subsystem is redundant.

NR(i, j) Number of redundant units for the j-th redundancy option for the i-th subsystem ($j \leq RO(i) + 1$).

R(i, j) ≡ Indicator equal to 0 or 1. If R(i, j) = 0 then the j-th redundancy option for subsystem i has operative redundancy. If R(i, j) = 1 then the j-th option is standby redundant ($j \leq NR(i, j) + 1$).

F(i, j) ≡ Mean operating time between failure for the j-th nonredundant option for subsystem i ($j \leq N(i) + 1$).

UC(i, j) ≡ Unit acquisition cost of the j-th nonredundant option for subsystem i ($j \leq N(i) + 1$).

CR(i, j) ≡ Average cost per repair of the j-th nonredundant option for subsystem i ($j \leq N(i) + 1$).

FR(i, j, k) ≡ Mean operating time between failure of the k-th redundant subsystem of the j-th redundancy option for subsystem i ($j \leq RO(i) + 1$; $k \leq NR(i, j)$).

UCR(i, j, k) ≡ Unit acquisition cost of the k-th redundant subsystem of the j-th redundancy option for subsystem i ($j \leq RO(i) + 1$, $k \leq NR(i, j)$).

CRR(i, j, k) ≡ Average cost per repair of the k-th redundant subsystem of the j-th redundancy option for subsystem i ($j \leq RO(i) + 1$, $k \leq NR(i, j)$).

2. OUTPUTS OF THE DSPC MODEL

The model outputs are printed in two tables. The Adjusted Baseline System is printed first to define those options which lead to a higher MCSP at lower cost (this results from the reliability management procedure described in Section III). The Adjusted Baseline System defines the starting point for the optimization procedure. The form of the printout for the Adjusted Baseline System is:

ADJUSTED BASELINE SYSTEM

Subsystem Number	Subsystem Name	Option for Adjusted Baseline
1		
2		
•		
•		
•		
NSSYS		

The meaning of the first two columns is self-evident. If, for subsystem i, the number ℓ appears in column 3 this means that nonredundant subsystem with $F(i, \ell)$, $UC(i, \ell)$, and $CR(i, \ell)$ should replace the baseline for subsystem i. If ℓR appears in column 3, then the baseline is replaced by that redundancy option corresponding to $FR(i, \ell, k)$, $UC(i, \ell, k)$, and $CRR(i, \ell, k)$ where $k = 1, 2, \ldots, NR(i, \ell)$. If $\ell = 1$ appears, then the baseline is the best starting point.

The second table presents the Optimal Subsystem Options in the following form:

OPTIMAL SUBSYSTEM OPTIONS

Configuration Identification (CI)	MCSP	Acquisition Cost	Logistic Support Cost	Total Cost	Subsystem Changed	Option Selected
Baseline CI = 1						
Adjusted Baseline CI = 2						
CI = 3						
•						
•						
•						

The configuration identification merely numbers the sequence of optimization steps. For the adjusted baseline system the corresponding options were defined and printed. For each configuration identification (after the baseline) the last two columns define the subsystem changed and the option selected for that subsystem.

3. DSPC CARD INPUTS

A description of the input cards for the DSPC model is presented in this section. As mentioned in Appendix A, the form of the card inputs to the MCSP model are identical to those for the DSPC model with the few exceptions described previously. It must be pointed out that on all numeric cards each value must be followed by a decimal point; on alphanumeric cards no decimal point is allowed.

a. First Card.

FIELD	INPUT
1	NSYS
2	NSSYS
3	NPHASES
4	LASTP
5	NYEARS
6	NMPM

b. Second Card(s).

FIELD	INPUT
1	Name of subsystem 1
2	Name of subsystem 2
•	•
•	•
•	•
8	Name of subsystem 8

Input as many cards required to name all NSSYS subsystems. The number of cards required is

$$\left[\frac{NSSYS}{8}\right]^+$$

where the notation $[y]^+$ denotes the smallest integer greater than or equal to y.

c. <u>Cards for the One Dimensional Arrays.</u>

One dimensional arrays are required for the inputs TR(i), N(i), RO(i), and IND1(i). For each of these inputs i = 1, 2, ..., NSSYS. The first card of a one dimensional array contains the array mnemonic beginning in column 1, i.e., starting in column 1 one of the mnemonics TR, N, RO, or IND1 is printed. The next $\left[\frac{NSSYS}{8}\right]^+$ cards for each mnemonic are as follows:

FIELD	INPUT
1	value corresponding to subsystem 1
2	value corresponding to subsystem 2
•	•
•	•
•	•
8	value corresponding to subsystem 8 .

Continue until all values are defined for each mnemonic. The procedure is repeated for each of the four one dimensional arrays.

d. <u>Cards for the Two Dimensional Arrays.</u>

Two dimensional arrays are required for the inputs TO(i, j), PA(i, j), NR(i, j), R(i, j), F(i, j), UC(i, j), and CR(i, j). The first card of any two dimensional array contains the array mnemonic (TO, PA, NR, R, F, UC, or CR) beginning in column 1. For example, after the mnemonic TO the next set of cards (corresponding to subsystem 1) is as follows:

FIELD	INPUT
1	TO(1, 1)
2	TO(1, 2)
•	•
•	•
•	•
8	TO(1, 8)

Continue to input TO(1, j) until j reaches its maximum value (for the i-th subsystem the maximum value of j for TO and PA is j = NPHASES; for F, UC, and CR the maximum value of j is N(i) + 1 $\leq$ 6; for NR and R the maximum value of j is RO(i) + 1). The next set of cards for subsystem 2 are:

FIELD	INPUT
1	TO(2, 1)
2	TO(2, 2)
•	•
•	•
•	•
8	TO(2, 8)

Continue to input TO(2, j) until j reaches its maximum value. Continue the process until the values of TO(i, j) are input for i = 1, 2, ..., NSSYS.

The process is repeated for each two dimensional array corresponding to the mnemonics TO, PA, NR, R, F, UC, and CR.

e. <u>Cards for the Three Dimensional Arrays</u>.

Three dimensional arrays are required for FR(i, j, k), UCR(i, j, k), and CRR(i, j, k). For each of these, i runs from 1 to NSSYS, j from 1 to R(i) + 1 $\leq$ 6, and k from 1 to NR(i, j) $\leq$ 5. As before, the first card contains the mnemonic beginning in column 1. For example, after the card containing the mnemonic FR the cards are as follows:

	FIELD	INPUT
	1	FR(1, 1, 1)
	2	FR(1, 1, 2)
	⋮	⋮
	NR(1, 1)	FR(1, 1, NR(1, 1))
	1	FR(1, 2, 1)
	2	FR(1, 2, 2)
2nd Card	⋮	⋮
	NR(1, 2)	FR(1, 2, NR(1, 2))
	1	FR(1, R(1) + 1, 1)
	2	FR(1, R(1) + 1, 1)
R(1) + 1 Card	⋮	⋮
	NR(1, R(1) + 1)	FR(1, R(1) + 1, NR(1, R(1) + 1))

Repeat the same procedure for each subsystem where the i-th subsystem consists of $R(i) + 1 \leq 6$ cards.

4 - DSPC PROGRAM LISTING

```
      PROGRAM DSPC(INPUT,OUTPUT,TAPE60=INPUT)
C
C     INPUT THE ARRAYS
      CALL INPUT
C     NOW EXECUTE THE MAIN BODY
      CALL CONTROL
C     THATS ALL FOLKS.....
      END
```

```
      SUBROUTINE INPUT
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TR(40),TO(40,20),PA(40
     1,20),N(40),RO(40),IND1(40),NR(40,6),R(40,6),F(40,6),UC(40,6),CR(40
     2,6),FR(40,6,6),UCR(40,6,6),CRR(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),P(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PR(40,
     46),UCAR(40,6),UCSR(40,6),UCPR(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),TABL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),ID1(
     640),SSC(40),SOS(40),TC(40,6)
      DIMENSION CARDS(8)
C
C     READ THE FIRST CARD
C
      READ 32, CARDS
      NSYS=CARDS(1)$NSSYS=CARDS(2)$NPHASES=CARDS(3)
      LASTP=CARDS(4)$NYEARS=CARDS(5)$NMPM=CARDS(6)
      PRINT 33, NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM
      READ 35, (NAMES(I),I=1,NSSYS)
      PRINT 36, (NAMES(I),I=1,NSSYS)
C
C     BEGIN READING THE ARRAYS
C
1     READ 37, IAN
      IF (EOF(60)) 31,2
2     PRINT 34, IAN
      IF (IAN.EQ.4HTR  ) GO TO 3
      IF (IAN.EQ.4HTO  ) GO TO 5
      IF (IAN.EQ.4HPA  ) GO TO 7
      IF (IAN.EQ.4HN   ) GO TO 9
      IF (IAN.EQ.4HRO  ) GO TO 11
      IF (IAN.EQ.4HIND1) GO TO 13
      IF (IAN.EQ.4HNR  ) GO TO 15
      IF (IAN.EQ.4HR   ) GO TO 17
      IF (IAN.EQ.4HF   ) GO TO 19
      IF (IAN.EQ.4HUC  ) GO TO 21
      IF (IAN.EQ.4HCR  ) GO TO 23
      IF (IAN.EQ.4HFR  ) GO TO 25
      IF (IAN.EQ.4HUCR ) GO TO 27
      IF (IAN.EQ.4HCRR ) GO TO 29
C     ERROR ON CARD
      PRINT 38
      GO TO 1
C
C     TR       MISSION OPERATING TIME/TOTAL OPERATING TIME
C
3     DO 4 I=1,NSSYS,8
      PRINT 39, I
      CALL READCD (CARDS)
      DO 4 J=1,8
      K=I+J-1
      IF (K.GT.NSSYS) GO TO 4
```

```
      TR(K)=CARDS(J)
4     CONTINUE
      GO TO 1
C
C     TO       OPERATING TIME
C
5     DO 6 I=1,NSSYS
      DO 6 J=1,NPHASES,8
      PRINT 40, I,J
      CALL READCD (CARDS)
      DO 6 L=1,8
      K=L+J-1
      IF (K.GT.NPHASES) GO TO 6
      TO(I,K)=CARDS(L)
6     CONTINUE
      GO TO 1
C
C     PA       PROBABILITY OF MISSION ABORT
C
7     DO 8 I=1,NSSYS
      DO 8 J=1,NPHASES,8
      PRINT 41, I,J
      CALL READCD (CARDS)
      DO 8 L=1,8
      K=L+J-1
      IF (K.GT.NPHASES) GO TO 8
      PA(I,K)=CARDS(L)
8     CONTINUE
      GO TO 1
C
C     N        NUMBER OF NON-REDUNDANT OPTIONS
C
9     DO 10 I=1,NSSYS,8
      PRINT 42, I
      CALL READCD (CARDS)
      DO 10 J=1,8
      K=I+J-1
      IF (K.GT.NSSYS) GO TO 10
      N(K)=CARDS(J)
10    CONTINUE
      GO TO 1
C
C     RO       NUMBER OF REDUNDANT OPTIONS
C
11    DO 12 I=1,NSSYS,8
      PRINT 43, I
      CALL READCD (CARDS)
      DO 12 J=1,8
      K=I+J-1
      IF (K.GT.NSSYS) GO TO 12
```

```
      RO(K)=CARDS(J)
12    CONTINUE
      GO TO 1
C
C     IND1      INDICATOR FOR BASELINE REDUNDANCY
C
13    DO 14 I=1,NSSYS,8
      PRINT 44, I
      CALL READCD (CARDS)
      DO 14 J=1,8
      K=I+J-1
      IF (K.GT.NSSYS) GO TO 14
      IND1(K)=CARDS(J)
14    CONTINUE
      GO TO 1
C
C     NR        NUMBER OF REDUNDANT SUBSYSTEMS
C
15    DO 16 I=1,NSSYS
      NRO=RO(I)+1
      PRINT 45, I
      CALL READCD (CARDS)
      DO 16 J=1,NRO
      NR(I,J)=CARDS(J)
16    CONTINUE
      GO TO 1
C
C     R         OPERATIVE OR STANDBY REDUNDANCY
C
17    DO 18 I=1,NSSYS
      NRO=RO(I)+1
      PRINT 46, I
      CALL READCD (CARDS)
      DO 18 J=1,NRO
      R(I,J)=CARDS(J)
18    CONTINUE
      GO TO 1
C
C     F         MTBF FOR NON-REDUNDANT SUBSYSTEMS
C
19    DO 20 I=1,NSSYS
      NNRO=N(I)+1
      PRINT 47, I
      CALL READCD (CARDS)
      DO 20 J=1,NNRO
      F(I,J)=CARDS(J)
20    CONTINUE
      GO TO 1
C
C     UC        UNIT COST FOR NON-REDUNDANT SUBSYSTEMS
```

SUBROUTINE INPUT

```
C
21      DO 22 I=1,NSSYS
        NNPO=N(I)+1
        PRINT 48, I
        CALL READCD (CARDS)
        DO 22 J=1,NNPO
        UC(I,J)=CARDS(J)
22      CONTINUE
        GO TO 1
C
C       CR        AVERAGE COST OF REPAIR FOR NON-REDUNDANT SUBSYSTEMS
C
23      DO 24 I=1,NSSYS
        NNPO=N(I)+1
        PRINT 49, I
        CALL READCD (CARDS)
        DO 24 J=1,NNPO
        CR(I,J)=CARDS(J)
24      CONTINUE
        GO TO 1
C
C       FR        MTBF FOR REDUNDANT SUBSYSTEMS
C
25      DO 26 I=1,NSSYS
        NRO=RO(I)+1
        DO 26 J=1,NRO
        NRS=NP(I,J)
        PRINT 50, I,J
        CALL READCD (CARDS)
        DO 26 K=1,NRS
        FR(I,J,K)=CARDS(K)
26      CONTINUE
        GO TO 1
C
C       UCR       UNIT COST FOR REDUNDANT SUBSYSTEMS
C
27      DO 28 I=1,NSSYS
        NRO=RO(I)+1
        DO 28 J=1,NRO
        NRS=NP(I,J)
        PRINT 51, I,J
        CALL READCD (CARDS)
        DO 28 K=1,NRS
        UCR(I,J,K)=CARDS(K)
28      CONTINUE
        GO TO 1
C
C       CRR       AVERAGE COST OF REPAIR FOR REDUNDANT SUBSYSTEMS
C
29      DO 30 I=1,NSSYS
```

```
      NRO=RO(I)+1
      DO 30 J=1,NRO
      NRS=NP(I,J)
      PRINT 52, I,J
      CALL READCD (CARDS)
      DO 30 K=1,NRS
      CRR(I,J,K)=CARDS(K)
30    CONTINUE
      GO TO 1
C     THATS ALL OF THE INPUT CARDS
31    CONTINUE
      RETURN
C
C
32    FORMAT (8F10.0)
33    FORMAT (14H1 FLEET SIZE =,I5,5X,22HNUMBER OF SUBSYSTEMS =,I5,5X,26
     1HNUMBER OF PHASES/MISSION =,I5/14H  LAST PHASE =,I5,5X,18HLIFE SPA
     2N(YEARS) =,I5,5X,33HNUMBER OF MISSIONS/MONTH/SYSTEM =,I5)
34    FORMAT (/,17H  NEW ARRAY ID = ,A4)
35    FORMAT (8A10)
36    FORMAT (/,16H SUBSYSTEM NAMES/,5(10(3X,A10)/))
37    FORMAT (A4)
38    FORMAT (//,26H ERROR ON ARRAY TYPE CARD.)
39    FORMAT (4H TR(,I3,4H) = )
40    FORMAT (4H TO(,I3,1H,,I2,4H) = )
41    FORMAT (4H PA(,I3,1H,,I2,4H) = )
42    FORMAT (3H N(,I3,4H) = )
43    FORMAT (4H RO(,I3,4H) = )
44    FORMAT (6H IND1(,I3,4H) = )
45    FORMAT (4H NP(,I3,7H, 1) = )
46    FORMAT (3H R(,I3,7H, 1) = )
47    FORMAT (3H F(,I3,7H, 1) = )
48    FORMAT (4H UC(,I3,7H, 1) = )
49    FORMAT (4H CR(,I3,7H, 1) = )
50    FORMAT (4H FP(,I3,1H,,I2,7H, 1) = )
51    FORMAT (5H UCR(,I3,1H,,I2,7H, 1) = )
52    FORMAT (5H CRR(,I3,1H,,I2,7H, 1) = )
      END
```

```
      SUBROUTINE READCD (CARDS)
      DIMENSION CARDS(8), ICARD(8)
      DATA (IB=10H          )
      READ 4, ICARD
      IF (EOF(60)) 1,2
1     PRINT 5
      CALL EXIT
C     CHECK TO SEE WHICH IS THE LAST NON-BLANK WORD
2     DO 3 I=1,8
      IF (ICARD(I).EQ.IB) GO TO 3
      IT=I
3     CARDS(I)=0.
      NC=IT*10
      DECODE (NC,6,ICARD) (CARDS(I),I=1,IT)
      PRINT 7, (CARDS(I),I=1,IT)
      RETURN
C
C
4     FORMAT (8A10)
5     FORMAT (//,61H  END-OF-FILE READ INSTEAD OF PARAMETER CARD. JOB AB
     1ORTED....)
6     FORMAT (8F10.2)
7     FORMAT (1H+,20X,8F10.2)
      END
```

```
      FUNCTION F1 (I,J)
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TR(40),TO(40,20),PA(40
     1,20),N(40),RO(40),IND1(40),NR(40,6),R(40,6),F(40,6),UC(40,6),CR(40
     2,6),FR(40,6,6),UCR(40,6,6),CRR(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),P(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PR(40,
     46),UCAR(40,6),UCSR(40,6),UCRP(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),IAPL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),ID1(
     640),SSC(40),SOS(40),TC(40,6)
      F1=EXP(-TP(I)/F(I,J))
      RETURN
      END
```

```
      FUNCTION F2 (I,J)
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TR(40),TO(40,20),PA(40
     1,20),M(40),RO(40),IND1(40),NR(40,6),P(40,6),F(40,6),UC(40,6),CR(40
     2,6),FF(40,6,6),UCP(40,6,6),CPR(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),R(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PR(40,
     46),UCAR(40,6),UCSR(40,6),UCPR(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),TABL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),IDI(
     640),SSC(40),SOS(40),TC(40,6)
      TEMP=1.
      NRO=NR(I,J)
      DO 1 L=1,NRO
    1 TEMP=TEMP*(1.-EXP(-TP(I)/FF(I,J,L)))
      F2=1.-TEMP
      RETURN
      END
```

```
      FUNCTION F3 (I,J)
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TR(40),TD(40,20),PA(40
     1,20),M(40),RO(40),IND1(40),NR(40,6),R(40,6),F(40,6),UC(40,6),CR(40
     2,6),FR(40,6,6),UCR(40,6,6),CRR(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),P(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PH(40,
     46),UCAR(40,6),UCSR(40,6),UCRP(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),TABL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),ID1(
     640),SSC(40),SOS(40),TC(40,6)
      TEMP=TP(I)/FR(I,J,1)
      T1=EXP(-TEMP)
      SUM=0.
      NRO=NR(I,J)
      DO 1 L=1,NRO
    1 SUM=SUM+((TEMP)**(L-1))/IFAC(L-1)
      F3=T1*SUM
      RETURN
      END
```

```
      FUNCTION IFAC (I)
C     COMPUTES THE FACTORIAL
      ITEMP=1
      IF (I.EQ.0) 1,2
1     IFAC=1
      RETURN
2     DO 3 K=1,I
3     ITEMP=ITEMP*K
      IFAC=ITEMP
      RETURN
      END
```

```
      FUNCTION F4 (I,J)
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TR(40),TO(40,20),PA(40
     1,20),N(40),RO(40),IND1(40),NR(40,6),P(40,6),F(40,6),UC(40,6),CR(40
     2,6),FR(40,6,6),UCP(40,6,6),CRP(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),P(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PR(40,
     46),UCAR(40,6),UCSR(40,6),UCPR(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),IABL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),ID1(
     640),SSC(40),SOS(40),TC(40,6)
      DIMENSION XY(5)
      SUM=0.
      T1=T(I)/FR(I,J,1)
      T2=CRP(I,J,1)
      T3=TNP(I)/FR(I,J,1)
      NRO=NR(I,J)-1
      DO 2 L=1,NRO
      LL=L
      SUM1=0.
      DO 1 K=1,LL
      T4=(T3**(K-1))/IFAC(K-1)
    1 SUM1=SUM1+T4
      XY(L)=SUM1*EXP(-T3)
    2 SUM=SUM-ALOG(XY(L))*CRR(I,J,L+1)
      F4=(T2+.5*SUM)*T1
      RETURN
      END
```

```
      FUNCTION G5 (I,J,M,XX)
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TH(40),TO(40,20),PA(40
     1,20),M(40),RO(40),IND1(40),NR(40,6),R(40,6),F(40,6),UC(40,6),CR(40
     2,6),FR(40,6,6),UCR(40,6,6),CRR(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),P(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PR(40,
     46),UCAR(40,6),UCSR(40,6),UCRR(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),IABL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),ID1(
     640),SSC(40),SOS(40),TC(40,6)
      IF (M.EQ.1) GO TO 3
      SUM=0.
      DO 2 L=1,M
      SUM1=1.
      DO 1 K=1,M
      IF (K.EQ.L) GO TO 1
      SUM1=SUM1*(FR(I,J,L)-FR(I,J,K))
1     CONTINUE
2     SUM=SUM+FR(I,J,L)**(M-1)*EXP(-XX/FR(I,J,L))/SUM1
      G5=SUM
      RETURN
3     G5=EXP(-XX/FR(I,J,1))
      RETURN
      END
```

```
      FUNCTION F5 (I,J)
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TR(40),TO(40,20),PA(40
     1,20),N(40),RO(40),IND1(40),NR(40,6),R(40,6),F(40,6),UC(40,6),CR(40
     2,6),FP(40,6,6),UCR(40,6,6),CPR(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),P(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PR(40,
     46),UCAR(40,6),UCSR(40,6),UCRP(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),TABL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),ID1(
     640),SSC(40),SOS(40),TC(40,6)
      M=NR(I,J)
      XX=TP(I)
      F5=G5(I,J,M,XX)
      RETURN
      END
```

```
      FUNCTION F6 (I,J)
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TR(40),TO(40,20),PA(40
     1,20),N(40),RO(40),IND1(40),NP(40,6),R(40,6),F(40,6),UC(40,6),CR(40
     2,6),FP(40,6,6),UCR(40,6,6),CRR(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),P(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PR(40,
     46),UCAR(40,6),UCSR(40,6),UCRR(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),TABL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),ID1(
     640),SSC(40),SOS(40),TC(40,6)
      SUM=0.
      NRO=NP(I,J)
      XX=TNP(I)
      DO 1 M=2,NRO
    1 SUM=SUM-ALOG(G5(I,J,M-1,XX))*CRR(I,J,M)/FP(I,J,M)
      F6=T(I)*(CRR(I,J,1)/FP(I,J,1)+.5*SUM)
      RETURN
      END
```

```
      SUBROUTINE CONTROL
      INTEGER SOS
      DIMENSION XTEMP(12)
      COMMON NSYS,NSSYS,NPHASES,LASTP,NYEARS,NMPM,TR(40),TO(40,20),PA(40
     1,20),N(40),RO(40),IND1(40),NR(40,6),P(40,6),F(40,6),UC(40,6),CR(40
     2,6),FR(40,6,6),UCR(40,6,6),CRR(40,6,6),NAMES(40),T(40),TP(40),TNP(
     340),P(40,6),UCA(40,6),UCS(40,6),PMC(40),CA(40),CS(40),C(40),PR(40,
     46),UCAR(40,6),UCSR(40,6),UCRR(40,6),X(40,20),IX(40,20),Y(40,20),Z(
     540,20),MY(40),TABL(40),XS(40),YS(40),ZS(40),XL(40),LAMBDA(40),ID1(
     640),SSC(40),SOS(40),TC(40,6)
C
C     PART I.  BASELINE MCSP
C
      DO 11 I=1,NSSYS
      T1=12.*NMPM*NYEARS*TR(I)
      SUM=0.
      DO 1 J=1,NPHASES
    1 SUM=SUM+TO(I,J)
      T(I)=T1*SUM
      SUM=0.
      DO 2 J=1,LASTP
    2 SUM=SUM+TO(I,J)*PA(I,J)
      TP(I)=SUM
      SUM=0.
      DO 3 J=1,NPHASES
    3 SUM=SUM+TO(I,J)*PA(I,J)
      TNP(I)=SUM
      IF (IND1(I).EQ.1) GO TO 4
C
C     NON-REDUNDANT SUBSYSTEM
C
      P(I,1)=F1(I,1)
      UCA(I,1)=UC(I,1)
      UCS(I,1)=T(I)*CR(I,1)/F(I,1)
      TC(I,1)=UCA(I,1)+UCS(I,1)
      GO TO 11
    4 IF (R(I,1).EQ.1.) GO TO 7
C
C     OPERATIVE REDUNDANCY
C
      P(I,1)=F2(I,1)
      UCA(I,1)=0.
      NRO=NR(I,1)
      DO 5 L=1,NRO
    5 UCA(I,1)=UCA(I,1)+UCR(I,1,L)
      SUM=0.
      DO 6 L=1,NRO
    6 SUM=SUM+T(I)*CRR(I,1,L)/FR(I,1,L)
      UCS(I,1)=SUM
      TC(I,1)=UCA(I,1)+UCS(I,1)
```

```
      GO TO 11
7     IF (FR(I,1,1).NE.FR(I,1,2)) GO TO 9
C
C     STANDBY REDUNDANCY  -  ALL MTBF ARE EQUAL
C
      P(I,1)=F3(I,1)
      NRO=NR(I,1)
      SUM=0.
      DO 8 L=1,NRO
8     SUM=SUM+UCR(I,1,L)
      UCA(I,1)=SUM
      UCS(I,1)=F4(I,1)
      TC(I,1)=UCA(I,1)+UCS(I,1)
      GO TO 11
C
C     STANDBY REDUNDANCY  -  ALL MTBF DIFFERENT
C
9     P(I,1)=F5(I,1)
      NRO=NR(I,1)
      UCA(I,1)=0.
      DO 10 L=1,NRO
10    UCA(I,1)=UCA(I,1)+UCR(I,1,L)
      UCS(I,1)=F6(I,1)
      TC(I,1)=UCA(I,1)+UCS(I,1)
11    CONTINUE
      PMC(1)=1.
      CA(1)=CS(1)=0.
      DO 12 I=1,NSSYS
      PMC(1)=PMC(1)*P(I,1)
      CA(1)=CA(1)+UCA(I,1)
12    CS(1)=CS(1)+UCS(I,1)
      CA(1)=CA(1)*NSYS
      CS(1)=CS(1)*NSYS
      C(1)=CA(1)+CS(1)
C
C     PART II.  SUCCESS PROBABILITY FOR EACH OPTION
C
      DO 22 I=1,NSSYS
      IF (N(I).EQ.0.AND.RO(I).EQ.0.) GO TO 22
      IF (N(I).EQ.0) GO TO 14
      NT=N(I)+1
      DO 13 J=2,NT
C
C     NON-REDUNDANT OPTIONS
C
      P(I,J)=F1(I,J)
      UCA(I,J)=UC(I,J)
      UCS(I,J)=T(I)*CR(I,J)/F(I,J)
13    TC(I,J)=UCA(I,J)+UCS(I,J)
14    IF (RO(I).EQ.0.) GO TO 22
```

```
C
C       REDUNDANT OPTIONS
C
        NRO=RO(I)+1.
        DO 21 J=2,NRO
        IF (R(I,J).EQ.1.) GO TO 17
C
C       OPERATIVE REDUNDANCY
C
        PR(I,J)=F2(I,J)
        SUM=0.
        NRI=NP(I,J)
        DO 15 L=1,NRI
15      SUM=SUM+UCR(I,J,L)
        UCAR(I,J)=SUM
        SUM=0.
        DO 16 L=1,NRI
16      SUM=SUM+T(I)*CRR(I,J,L)/FR(I,J,L)
        UCSR(I,J)=SUM
        UCRR(I,J)=UCAR(I,J)+UCSR(I,J)
        GO TO 21
C
C       STANDBY REDUNDANCY
C
17      IF (FR(I,J,1).NE.FR(I,J,2)) GO TO 19
C
C       EQUAL MTBF
C
        PR(I,J)=F3(I,J)
        NRI=NP(I,J)
        SUM=0.
        DO 18 L=1,NRI
18      SUM=SUM+UCR(I,J,L)
        UCAR(I,J)=SUM
        UCSR(I,J)=F4(I,J)
        UCRR(I,J)=UCAR(I,J)+UCSR(I,J)
        GO TO 21
C
C       UNEQUAL MTBF
C
19      PR(I,J)=F5(I,J)
        NRI=NP(I,J)
        SUM=0.
        DO 20 L=1,NRI
20      SUM=SUM+UCR(I,J,L)
        UCAR(I,J)=SUM
        UCSR(I,J)=F6(I,J)
        UCRR(I,J)=UCAR(I,J)+UCSR(I,J)
21      CONTINUE
22      CONTINUE
```

```
C
C     PART III.     ADJUSTED BASELINE SYSTEM
C
      DO 30 I=1,NSSYS
      IF (N(I).EQ.0.AND.RO(I).EQ.0.) GO TO 30
      IN=N(I)+1
      IR=RO(I)+1
      INF=1$IRF=2
C     PUT THE ARRAYS INTO A TEMPORARY ARRAY
      DO 23 J=INF,IN
23    XTEMP(J)=P(I,J)
      DO 24 J=IRF,IR
24    XTEMP(IN+J-1)=PR(I,J)
      JX=IN+IR-1
      CALL TSORT (XTEMP,JX)
C     NOW PLACE INTO X
      DO 28 J=1,JX
      X(I,J)=XTEMP(J)
      DO 26 L=INF,IN
      IF (P(I,L).EQ.XTEMP(J)) 25,26
25    IX(I,J)=L+33B
      IX(I,J)=SHIFT(IX(I,J),6)+55B
      Y(I,J)=TC(I,L)
      Z(I,J)=UCA(I,L)
26    CONTINUE
      DO 28 L=IRF,IR
      IF (PR(I,L).EQ.XTEMP(J)) 27,28
27    IX(I,J)=L+33B
      IX(I,J)=SHIFT(IX(I,J),6)+22B
      Y(I,J)=UCRR(I,L)
      Z(I,J)=UCAR(I,L)
28    CONTINUE
      YMIN=1.E+300
C     DETERMINE THE MINIMUM OF Y
      DO 29 J=1,JX
      IF (Y(I,J).GT.YMIN) GO TO 29
      IF (X(I,J).LT.P(I,1)) GO TO 29
      YMIN=Y(I,J)
      MY(I)=J
29    CONTINUE
      IABL(I)=IX(I,MY(I))
      XS(I)=X(I,MY(I))
      YS(I)=Y(I,MY(I))
      ZS(I)=Z(I,MY(I))
30    CONTINUE
      PMC(2)=1.
      DO 31 I=1,NSSYS
31    PMC(2)=PMC(2)*XS(I)
      CA(2)=C(2)=0.
      DO 32 I=1,NSSYS
```

```
      CA(2)=CA(2)+ZS(I)
32    C(2)=C(2)+YS(I)
      CA(2)=CA(2)*NSYS
      C(2)=NSYS*C(2)
      CS(2)=C(2)-CA(2)
C
C     PART IV.   OPTIMAL OPTIONS
C
      DO 35 I=1,NSSYS
      JX=N(I)+RO(I)+1
      IF (MY(I).NE.JX) GO TO 33
      XL(I)=0.
      GO TO 35
33    YMAX=-1.E+300
      IF=MY(I)$IF1=IF+1
      DO 34 J=IF1,JX
      DC=Y(I,J)-Y(I,IF)
      YM=(X(I,J)/X(I,IF))**(1./DC)
      IF (YM.LT.YMAX) GO TO 34
      LAMBDA(I)=J
      YMAX=YM
34    CONTINUE
      DC=Y(I,LAMBDA(I))-Y(I,IF)
      XL(I)=(X(I,LAMBDA(I))/X(I,IF))**(1./DC)
35    CONTINUE
      K=2
36    K=K+1
      DO 37 I=1,NSSYS
      IF (XL(I).NE.0.) GO TO 38
37    CONTINUE
      MAX=K-1
      GO TO 42
38    YMAX=-1.E+300
      DO 39 I=1,NSSYS
      IF (XL(I).LT.YMAX) GO TO 39
      ID1(K)=I
      YMAX=XL(I)
39    CONTINUE
      IA=ID1(K)
      IB=LAMBDA(IA)
      ID=MY(IA)
      PMC(K)=PMC(K-1)*(X(IA,IB)/X(IA,ID))
      CA(K)=CA(K-1)+NSYS*(Z(IA,IB)-Z(IA,ID))
      C(K)=C(K-1)+NSYS*(Y(IA,IB)-Y(IA,ID))
      CS(K)=C(K)-CA(K)
      SSC(K)=IA
      SOS(K)=IX(IA,IB)
      MY(IA)=LAMBDA(IA)
      IJ=N(IA)+RO(IA)+1
      IF (MY(IA).NE.IJ) GO TO 40
```

```
      XL(IA)=0.
      GO TO 36
   40 YMAX=-1.E+300
      IF=MY(IA)$IF1=IF+1
      DO 41 J=IF1,TJ
      DC=Y(IA,J)-Y(IA,IF)
      YM=(X(IA,J)/X(IA,IF))**(1./DC)
      IF (YM.LT.YMAX) GO TO 41
      LAMBDA(IA)=J
      YMAX=YM
   41 CONTINUE
      DC=Y(IA,LAMBDA(IA))-Y(IA,IF)
      XL(IA)=(X(IA,LAMBDA(IA))/X(IA,IF))**(1./DC)
      GO TO 36
C
C     FINISHED.  NOW PRINT
C
C     ADJUSTED BASELINE SYSTEM
C
   42 PRINT 46
      DO 43 I=1,NSSYS
      PRINT 47, I,NAMES(I),IABL(I)
   43 CONTINUE
C
C     OPTIONS
C
      PRINT 48
      PRINT 49, PMC(1),CA(1),CS(1),C(1)
      PRINT 50, PMC(2),CA(2),CS(2),C(2)
      IF (MAX.LT.3) GO TO 45
      DO 44 I=3,MAX
      PRINT 51, I,PMC(I),CA(I),CS(I),C(I),SSC(I),SOS(I)
   44 CONTINUE
   45 CONTINUE
      RETURN
C
C
   46 FORMAT (1H1,9X,24HADJUSTED BASELINE SYSTEM//,2X,9HSUBSYSTEM,5X,9HS
     1UBSYSTEM,5X,19HOPTION FOR ADJUSTED/,3X,6HNUMBER,9X,4HNAME,13X,8HBA
     2SELINE)
   47 FORMAT (1X,I8,6X,A10,14X,R2)
   48 FORMAT (1H1,43X,25HOPTIMAL SUBSYSTEM OPTIONS,//,2X,13HCONFIGURATIO
     1N,17X,11HACQUISITION,7X,8HLOGISTIC,9X,5HTOTAL,7X,9HSUBSYSTEM,7X,6H
     2OPTION/,2X,14HIDENTIFICATION,6X,4HMCSP,10X,4HCOST,8X,12HSUPPORT CO
     3ST,8X,4HCOST,8X,7HCHANGED,7X,8HSELECTED)
   49 FORMAT (/,16H   BASELINE, CI=1,3X,F6.3,6X,F10.2,8X,F10.2,4X,F10.2)
   50 FORMAT (/,16H   ADJUSTED, CI=2,/,10H   BASELINE,9X,F6.3,6X,F10.2,8X,
     1F10.2,4X,F10.2)
   51 FORMAT (/,4X,3HCI=,I4,8X,F6.3,6X,F10.2,8X,F10.2,4X,F10.2,7X,F5.0,9
     1X,R2)
```

```
      END                                                              L 301-
```

```
      SUBROUTINE TSORT (A,N)
      DIMENSION A(1), IL(16), IU(16)
      I=1
      J=N
      M=0
1     IF (J.LE.I) GO TO 9
2     IJ=(I+J)/2
      K=I
      L=J
      IF (A(I).LE.A(J)) GO TO 3
      T=A(J)
      A(J)=A(I)
      A(I)=T
3     T=A(IJ)
      IF (A(I).LE.T) GO TO 4
      A(IJ)=A(I)
      A(I)=T
      T=A(IJ)
      GO TO 5
4     IF (T.LE.A(J)) GO TO 5
      A(IJ)=A(J)
      A(J)=T
      T=A(IJ)
5     L=L-1
      IF (T.LT.A(L)) GO TO 5
      TT=A(L)
6     K=K+1
      IF (A(K).LT.T) GO TO 6
      IF (L.LT.K) GO TO 7
      A(L)=A(K)
      A(K)=TT
      GO TO 5
7     M=M+1
      IF (L-I.LE.J-K) GO TO 8
      IL(M)=I
      IU(M)=L
      I=K
      GO TO 10
8     IL(M)=K
      IU(M)=J
      J=L
      GO TO 10
9     IF (M.EQ.0) RETURN
      I=IL(M)
      J=IU(M)
      M=M-1
10    IF (J-I.GE.13) GO TO 2
      IF (I.EQ.1) GO TO 1
11    I=I+1
      IF (J.LT.I) GO TO 9
```

SUBROUTINE TSORT (A,N)

```
      T=A(I)
      IF (A(I-1).LE.T) GO TO 11
      K=I-1
12    A(K+1)=A(K)
      K=K-1
      IF (T.LT.A(K)) GO TO 12
      A(K+1)=T
      GO TO 11
      END
```

LIST OF SYMBOLS

A ≡ Availability of an aircraft.

α_i ≡ Ratio of total operating time to mission operating time.

C_a ≡ Unit acquisition cost of a redundant unit.

C_{ij} ≡ Cost of the j-th option for the i-th subsystem ($j = 1, 2, \ldots, n(i)$; $i = 1, 2, \ldots, N_s$)

C_s ≡ Logistic support cost of a single redundant unit.

CR_i ≡ Average cost per repair of the i-th subsystem.

CR_{ij} ≡ Average cost per repair associated with the j-th option for the i-th subsystem.

f_i ≡ The i-th failure mode of a certain subsystem.

LSC_i ≡ Average yearly logistic support cost of the i-th subsystem.

LSC_y ≡ Total system logistic support cost during y years.

m ≡ Average number of missions per month per system.

N ≡ Total number of systems (fleet size).

N_p ≡ Number of mission phases.

N_s ≡ Total number of subsystems.

$n(i)$ ≡ Number of options for the i-th subsystem.

P_a ≡ Probability of abort given a failure of a certain subsystem.

P_{aij} ≡ Conditional probability of mission abort given that the i-th subsystem fails during the j-th mission phase.

P_{as} ≡ Conditional probability of abort due to safety factors given a failure.

P_{apj} ≡ Conditional probability of abort during phase j given no abort before phase j.

P_{ai} ≡ Probability that a failure of the i-th redundant unit is an abort causing failure.

P_c = Probability aircraft reaches target and releases weapons without an abort causing failure given that it survives.

$P_c(n, T)$ $\equiv$ Probability that a subsystem with n redundant units will not cause an abort during operating time T.

$P_{c\ell}$ $\equiv$ Probability that the system completes the ℓ-th mission phase without an abort causing failure.

P_E $\equiv$ Conditional probability of reduced effectiveness given a failure.

P_i $\equiv$ Probability that the i-th subsystem completes its function without an abort causing failure.

P_{ic} $\equiv$ Probability that the i-th subsystem completes its function (i.e., operates for time $\sum_{j=1}^{N_p} t_{ij}$).

P_{mc} $\equiv$ Probability that a mission is completed without an abort causing failure.

P_s = Single sortie survival probability.

P_{sa} = Probability aircraft aborts before releasing weapons and survives the return trip.

P_{s1} = Probability aircraft survives to release its weapons on target.

P_{s2} = Probability aircraft survives return trip after weapons are released.

R_i $\equiv$ Expected number of repairs of the i-th subsystem during one year.

ρ = "Kill Potential" = expected number of targets destroyed after aircraft reaches the target area.

S = Number of sorties aircraft flies (if it survives).

T $\equiv$ Operating time of a certain subsystem.

LIST OF SYMBOLS (continued)

$T(S)$ = Expected number of targets destroyed after S sorties.

T_K = Expected number of targets destroyed during the "lifetime" of the aircraft, i.e., $S \to \infty$.

t_{ij} ≡ Operating time of the i-th subsystem during the j-th mission phase.

T_i ≡ Total y-year operating time of subsystem i.

t_i ≡ Operating time of i-th subsystem ($i = 1, 2, \ldots, N_s$) during one mission, i.e., duty cycle of i-th subsystem.

T_m ≡ Aircraft mission time.

t_r ≡ Mean time to restore.

τ ≡ Mean operating time between failures for a certain subsystem. For the discussion of failure modes a subscript on this symbol would unnecessarily complicate the development.

τ_a ≡ Mean operating time between abort causing failures.

τ_{ai} ≡ Mean operating time between abort type failures of the i-th standby redundant unit.

τ_{ar} ≡ Mean operating time between abort type failures of a redundant unit.

τ_i ≡ Mean operating time between failures of the i-th subsystem.

τ_{ri} ≡ Mean operating time between failures of the i-th redundant unit.

τ_s ≡ MTBF of the total aircraft system.

τ_{ij} ≡ Lower MTBF for the j-th option for the i-th subsystem.

$\bar{\tau}_{ij}$ ≡ Upper MTBF for the j-th option for the i-th subsystem.

y ≡ Number of years to be considered in the calculation of logistic support costs.